GEOMETRIYA

I BOB. FAZODA KOORDINATALAR SISTEMASI VA VEKTORLAR

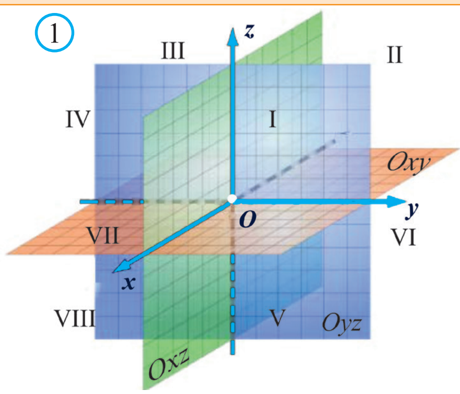
1. FAZODA KOORDINATALAR SISTEMASI

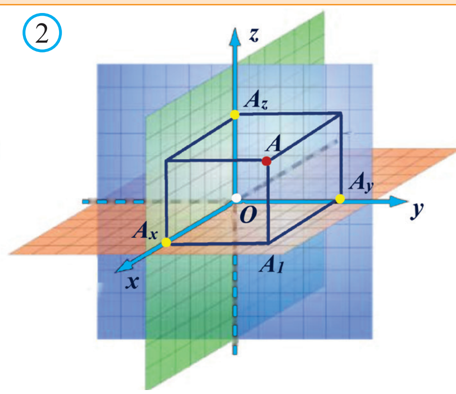
1.1. Fazoda dekart koordinatalar sistemasi

Tekislikda dekart koordinatalari sistemasi bilan quyi sinflarda tanishgansiz. Fazoda koordinatalar sistemasi ham tekislikdagiga o‘xshash kiritiladi. *O* nuqtada kesishuvchi va koordinata boshi shu nuqtada bo‘lgan o‘zaro perpendikular uchta *Ox*, *Oy* va *Oz* koordinata o‘qlarini qaraymiz.

Bu to‘g‘ri chiziqlarning har bir jufti orqali *Oxy*, *Oxz* va *Oyz* tekisliklar o‘tkazamiz (1- rasm). Fazoda to‘g‘ri burchakli dekart koordinatalari sistemasi shu tariqa kiritiladi va unda

O nuqta – *koordinatalar boshi*,
Ox, *Oy* va *Oz* to‘g‘ri chiziqlar – *koordinata o‘qlari*,
Ox – *abssissalar*, *Oy* – *ordinatalar* va *Oz* o‘qi – *applikatalar o‘qi*,
Oxy, *Oyz* va *Oxz* tekisliklar – *koordinatalar tekisliklari* deb ataladi.





Koordinatalar tekisliklari fazoni 8 ta *oktantaga* (*nimchorakka*) bo‘ladi (1- rasm).

Fazoda ixtiyoriy *A* nuqta berilgan bo‘lsin. Bu nuqtadan *Oxy*, *Oyz* va *Oxz* koordinata tekisliklariga perpendikular tekisliklar o‘tkazamiz (2- rasm). Bu tekisliklardan biri *Ox* o‘qini A_x nuqtada kesib o‘tadi.

A_x nuqtaning *x* o‘qidagi koordinatasi *A* nuqtaning *x* – *koordinatasi* yoki *abssissasi* deb ataladi.

A nuqtaning *y* – *koordinatasi* (*ordinatasi*) hamda *z* – *koordinatasi* (*applikatasi*) ham shu tariqa aniqlanadi.

A nuqtaning koordinatalari $A(x;\ y;\ z)$ yoki qisqaroq $(x;\ y;\ z)$ tarzda belgilanadi. 3- rasmda tasvirlangan nuqtalar quyidagi koordinatalarga ega: $A(0;\ 5;\ 0)$, $B(4;\ 0;\ 0)$, $M(0;\ 5;\ 4)$, $K(2;\ 3;\ 4)$, $P(-2;\ 3;\ -4)$.

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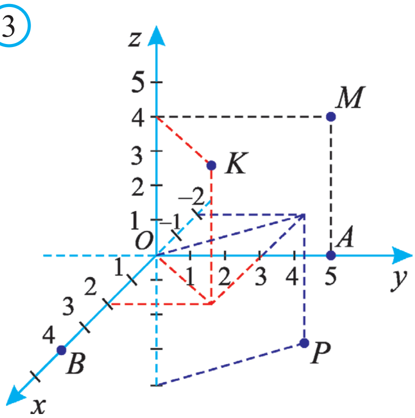

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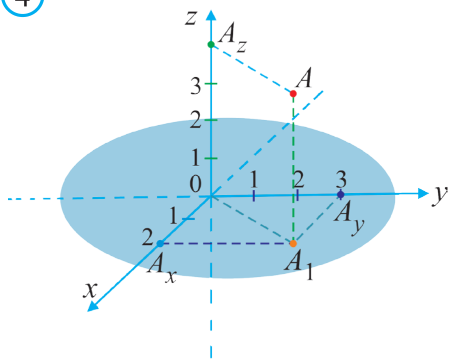

1- masala. Fazoda dekart koordinatalari sistemasi kiritilgan. Undagi $A(2;\ 3;\ 4)$ nuqtaning o‘rnini aniqlang.

Yechish. Koordinata boshidan *Ox* va *Oy* o‘qlarining musbat yo‘nalishida, mos ravishda, $OA_x = 2$ va $OA_y = 3$ kesmalarni qo‘yamiz (4- rasm).

A_x nuqtadan *Oxy* tekislikda yotgan va *Oy* o‘qiga parallel to‘g‘ri chiziq o‘tkazamiz. A_y nuqtadan *Oxy* tekislikda yotgan va *Ox* o‘qiga parallel to‘g‘ri chiziq o‘tkazamiz. Bu to‘g‘ri chiziqlar kesishish nuqtasini A_1 bilan belgilaymiz. A_1 nuqtadan *Oxy* tekislikka perpendikular o‘tkazamiz va unda *Oz* o‘qining musbat yo‘nalishida $AA_1 = 4$ kesma qo‘yamiz. Hosil bo‘lgan $A(2;\ 3;\ 4)$ nuqta izlanayotgan nuqta bo‘ladi. □

Zamonaviy raqamli-dasturli boshqariladigan stanoklar va avtomatlashtirilgan robotlar uchun koordinatalar sistemasidan foydalanib dasturlar tuziladi va ular asosida metallarga ishlov beriladi (5- rasm).

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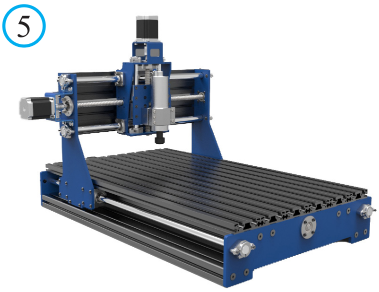

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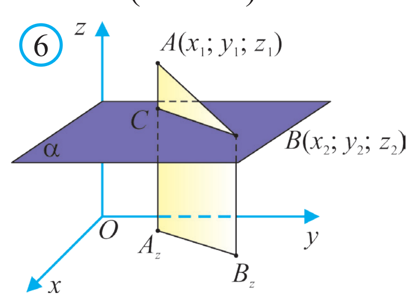

1.2. Ikki nuqta orasidagi masofa

Ikkita $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalar berilgan bo‘lsin.

1. Avval AB to‘g‘ri chiziq Oz o‘qiga parallel bo‘lmagan holni qaraymiz (6- rasm). A va B nuqtalar orqali Oz o‘qiga parallel chiziqlar o‘tkazamiz. Ular Oxy tekislikni A_z va B_z nuqtalarda kesib o‘tsin.

Bu nuqtalarning z koordinatasi 0 ga teng bo‘lib, x va y koordinatalari esa mos ravishda A, B nuqtalarning x va y koordinatalariga teng.

Endi B nuqta orqali Oxy tekislikka parallel α tekislik o‘tkazamiz. U AA_z to‘g‘ri chiziqni biror C nuqtada kesib o‘tadi.

Pifagor teoremasiga ko‘ra: $AB^2 = AC^2 + CB^2$.

Lekin $CB = A_zB_z$, $A_zB_z^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ va $AC = |z_2 - z_1|$.

Shuning uchun $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

2. AB kesma Oz o‘qiga parallel, ya’ni $AB = |z_2 - z_1|$ bo‘lganda ham yuqoridagi formula o‘rinli bo‘ladi, chunki bu holda $x_1 = x_2$, $y_1 = y_2$.

Demak, *A va B nuqtalar orasidagi masofa*:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (1)$$

Izoh. (1) formula to‘g‘ri burchakli parallelepipedning o‘lchamlari $a = |x_2 - x_1|$, $b = |y_2 - y_1|$, $c = |z_2 - z_1|$ bo‘lganda, uning diagonali uzunligini ifodalaydi.

Sfera va shar tenglamasi. Ma’lumki, $A(a; b; c)$ nuqtadan R masofada yotgan barcha $M(x; y; z)$ nuqtalar sferani tashkil qiladi (7- rasm). Unda (1) formulaga ko‘ra, markazi $A(a; b; c)$ nuqtada radiusi R ga teng bo‘lgan sferada yotgan barcha nuqtalar koordinatalari $(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$ tenglikni qanoatlantiradi.

Unda, ravshanki, markazi $A(a; b; c)$ nuqtada, radiusi R ga teng bo‘lgan shar tenglamasi $(x - a)^2 + (y - b)^2 + (z - c)^2 \leq R^2$ tarzda ifodalanadi.

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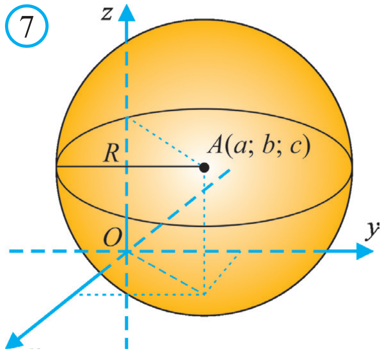

2- masala. Uchlari $A(9; 3; -5)$, $B(2; 10; -5)$, $C(2; 3; 2)$ nuqtalarda bo‘lgan ABC uchburchakning perimetrini toping.

Yechish: ABC uchburchakning perimetri $P = AB+AC+BC$. Ikki nuqta orasidagi masofa formulasi $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ dan foydalanib uchburchak tomonlarini topamiz:

$$AB=\sqrt{(2-9)^2+(10-3)^2+(-5+5)^2}=\sqrt{49+49}=7\sqrt{2},$$

$$AC=\sqrt{(2-9)^2+(3-3)^2+(2+5)^2}=\sqrt{49+49}=7\sqrt{2},$$

$$BC=\sqrt{(2-2)^2+(3-10)^2+(2+5)^2}=\sqrt{49+49}=7\sqrt{2}.$$

Demak, ABC uchburchak teng tomonli va uning perimetri:
$P=3\cdot 7\sqrt{2}=21\sqrt{2}$. *Javob:* $21\sqrt{2}$. □

1.3. Kesma o‘rtasining koordinatalari

$A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2;)$ – ixtiyoriy nuqtalar bo‘lib, AB kesmaning o‘rtasi $C(x; y; z)$ bo‘lsin (8- rasm).

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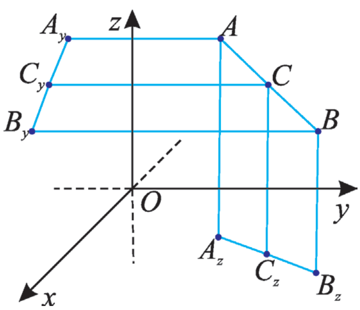

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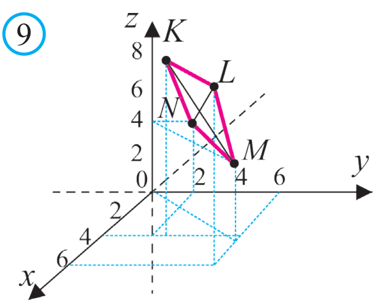

A, B va C nuqtalar orqali Oz o‘qiga parallel to‘g‘ri chiziqlar o‘tkazamiz. Ular Oxy tekislikni $A_z(x_1; y_1; 0)$, $B_z(x_2; y_2; 0)$ va $C_z(x; y; 0)$ nuqtalarda kesib o‘tsin.

Fales teoremasiga ko‘ra C_z nuqta A_zB_z kesmaning o‘rtasi bo‘ladi.

Unda tekislikda kesma o‘rtasining koordinatalarini topish formulasiga ko‘ra

$$x=\frac{x_1+x_2}{2},\quad y=\frac{y_1+y_2}{2}.$$

z ni topish uchun Oxy tekislik o‘rniga Oxz yoki Oyz tekislikni olish kifoya.

Bunda z uchun ham yuqoridagilarga o‘xshash formula hosil qilinadi.

$$x=\frac{x_1+x_2}{2},\quad y=\frac{y_1+y_2}{2}\quad z=\frac{z_1+z_2}{2}.$$

Shunga o‘xshash, berilgan AB kesmani λ nisbatda ($AP:PB=\lambda$) bo‘luvchi $P(x_1; y_1; z_1)$ nuqtaning koordinatalari A va B nuqtalarning koordinatalari orqali

$$x=\frac{x_1+\lambda x_2}{1+\lambda},\quad y=\frac{y_1+\lambda y_2}{1+\lambda},\quad z=\frac{z_1+\lambda z_2}{1+\lambda}$$

formulalar yordamida topiladi. Ularning to‘g‘riligini mustaqil ko‘rsating.

3-masala. Uchlari $M(3; 6; 4)$, $N(0; 2; 4)$, $K(3; 2; 8)$, $L(6; 6; 8)$ nuqtalarda bo‘lgan *MNKL* to‘rtburchakning parallelogramm ekanligini isbotlang (9- rasm).

Isbot: Masalani yechishda diagonallari kesishish nuqtasida teng ikkiga bo‘linadigan to‘rtburchakning parallelogramm ekanligidan foydalanamiz.

MK kesma o‘rtasining koordinatalari:

$$x=\frac{3+3}{2}=3; \qquad y=\frac{6+2}{2}=4; \qquad z=\frac{4+8}{2}=6.$$

NL kesma o‘rtasining koordinatalari:

$$x=\frac{0+6}{2}=3; \qquad y=\frac{2+6}{2}=4; \qquad z=\frac{4+8}{2}=6.$$

*MK*va *NL* kesmalar o‘rtalarining koordinatalari bir xil ekanini ko‘ramiz. Bu mazkur kesmalar kesishishini va kesishish nuqtasida ular teng ikkiga bo‘linishini bildiradi.

Demak, *MNLK* to‘rtburchak – parallelogramm. □

Mavzuga oid masalalar va amaliy topshiriqlar

1. 10- rasmda tasvirlangan nuqtalarning koordinatalarini aniqlang.

2. Fazoda dekart koordinatalari sistemasi kiritilgan bo‘lib, unda $A(0; 3; 1)$, $B(–2; 0; 0)$, $C(0; 0; 8)$, $D(0; –9; 0)$, $E(5; –1; 2)$, $F(–6; 2; 1)$ nuqtalar berilgan. Bu nuqtalar qaysi *a*) koordinatalar o‘qda; *b*) koordinatalar tekisligida; *c*) oktantda yotadi?

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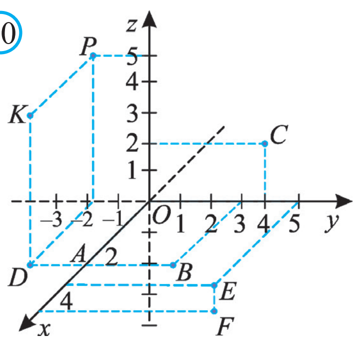

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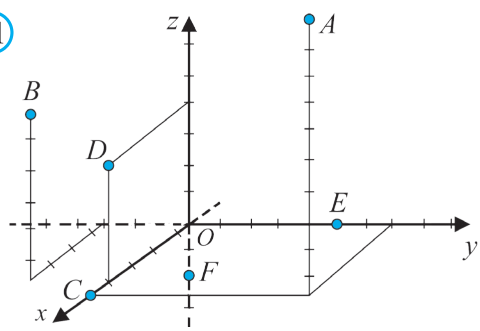

3. 11- rasmdagi nuqtalar koordinatalarini toping.

4. 12- rasmda belgilangan nuqtalarning koordinatalarini toping.

5. 13- rasmda diagonali $\sqrt{2}$ ga teng bo‘lgan kvadrat tasvirlangan. Uning uchlari koordinatalarini toping.

6. *A* (3; 2; 4) nuqtaning koordinata tekisliklaridagi proyeksiyasi koordinatalarini toping.

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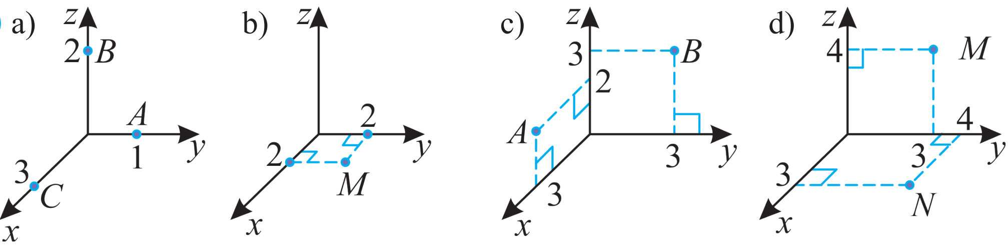

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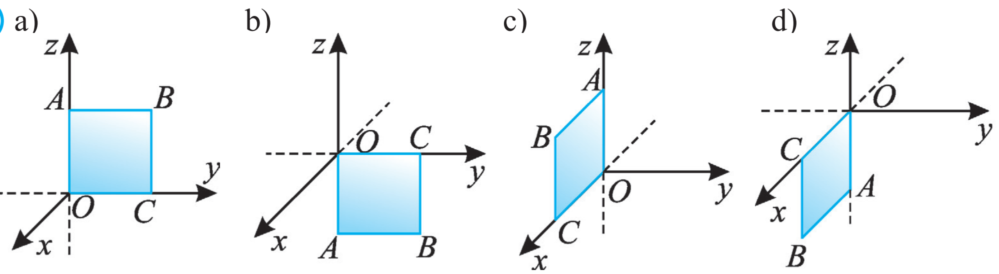

7. Fazoda dekart koordinatalari sistemasi kiritilgan bo‘lib, unda A (–1; 2; –3), B(0; 1; 2), C(0; 0; 5), D(–2; 2; 0), E(5; –1; 0), F(0; 2; 0), G(9; 0; 0), H(9; 0; 2), I(6; 3; 1), J(–6; 3; 5), K(–6; –2; 3), L(6; –2; 4), M(6; 3; –9), N(–6; 3; –8), O(–6; –3; –6), P(6; –3; –2) nuqtalar berilgan bo‘lsin. Bu nuqtalar qaysi koordinatalar o‘qida, kordinatalar tekisligida va oktantda yotadi? Quyida berilgan jadvalni berilgan namunalarga ko‘ra to‘ldiring.

Nuqta o‘rni	Nuqta koordinatalari xususiyati	Nuqta
Ox o‘qi	$y=0$, $z=0$ faqat x koordinata noldan farqli	G(9; 0; 0)
Oy o‘qi		
Oz o‘qi		
Oxz tekislik	$z=0$, x va y koordinatalar noldan farqli	D(–2; 2; 0)
Oyz tekislik		
Oxz tekislik		
1- oktant	$x>0$, $y>0$, $z>0$	I(6; 3; 1)
2- oktant		
3- oktant		
4- oktant		
5- oktant		
6- oktant		
7- oktant		
8- oktant		

8. $A(2; 0; -3)$ va $B(3; 4; 0)$ nuqtalar orasidagi masofani toping.

9. $A(3; 3; 3)$ nuqtadan a) koordinata tekisliklarigacha; b) koordinata o‘qlarigacha; c) koordinata boshigacha bo‘lgan masofalarni toping.

10. $M(2; -3; 1)$ nuqtadan koordinata tekisliklarigacha bo‘lgan masofalarni toping.

11. Koordinata tekisliklarining har biridan 3 birlik masofada uzoqlashgan nuqtaning o‘rnini aniqlang.

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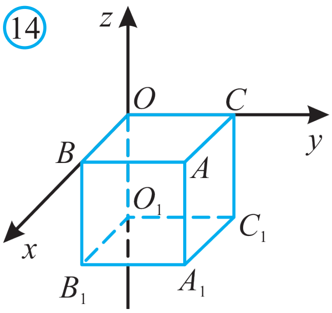

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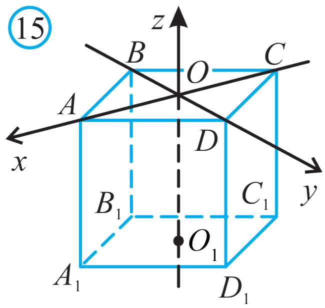

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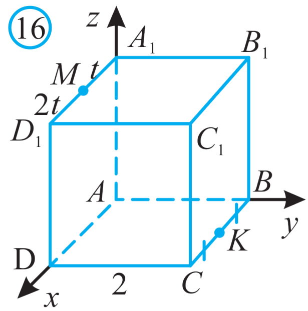


12. Agar $OA=2\sqrt{2}$ bo‘lsa, 14- rasmda tasvirlangan kubning uchlari koordinatalarini toping.

13. $C(2; 5; -1)$ va $D(2; 1; -6)$ nuqtalarning qaysi biri koordinata boshiga yaqin joylashgan?

14. Uchlari $A(1; 2; 3)$, $B(2; 3; 1)$, $C(3; 1; 2)$ nuqtalarda bo‘lgan uchburchakning perimetrini toping.

15. Uchlari $A(1; 2; 3)$, $B(2; 3; 4)$, $C(3; 4; 5)$ nuqtalarda bo‘lgan uchburchak mavjudmi?

16. $A(-2; 0; 5)$, $B(-1; 2; 3)$, $C(1; 1; -3)$, $D(0; -1; -1)$ nuqtalar parallelogramm uchlari ekanligini isbotlang.

17. ABC uchburchak turini aniqlang, uning perimetri va yuzini toping: a) $A(3; 0; 0)$, $B(0; 3; 0)$, $C(0; 0; 3)$; b) $A(2; 0; 5)$, $B(3; 4; 0)$, $C(2; 4; 0)$; c) $A(2; 4; -1)$, $B(-1; 1; 2)$, $C(5; 1; 2)$.

18. Oxy tekisligida yotuvchi va $A(0; 1; -1)$, $B(-1; 0; -1)$, $C(0; -1; 0)$ nuqtalardan baravar uzoqlikda yotuvchi nuqtaning koordinatalarini toping.

19. $A(1; 1; 1)$, $B(-1; 1; 1)$, $C(-1; -1; 1)$, $C_1(-1; -1; -1)$ nuqtalar $ABCDA_1B_1C_1D_1$ kubning uchlari bo‘lsa, uning qolgan uchlari koordinatalarini toping.

20. Uchlari $S(0; 0; 0)$, $A(2; 0; 0)$, $B(0; 2; 0)$, $C(0; 0; 2)$ nuqtalarda bo‘lgan $SABC$ piramidaning muntazam ekanligini isbotlang.

21. Markazi koordinatalar boshida, radiusi 5 ga teng bo‘lgan sfera va shar tenglamalarini yozing.

22. Markazi A (1; 2; 4) nuqtada, radiusi 3 ga teng bo‘lgan shar tenglamasini yozing.

23. Diametri uchlari A (–2; 1; 3), B (0; 2; 1) nuqtalarda yotgan sfera tenglamasini yozing.

24. Qalin qog‘ozdan kub modelini yasang. Uning bitta uchini koordinata boshi, undan chiquvchi qirralarni birlik ortlar sifatida olib, uning boshqa uchlari koordinatalarini toping.

25. AB kesma o‘rtasining koordinatalarini toping:
1) $A(-1; 0; 0)$, $B(1; 2; 0)$; 2) $A(0; 0; 0)$, $B(2; 2; 2)$; 3) $A(-2; 4; 2)$, $B(2; -4; 2)$,
4) $A(1{,}2; -3; 6{,}3)$, $B(-2{,}6; 3{,}2; -5{,}1)$; 5) $A(\sqrt{3}; 2; 1-\sqrt{2})$, $B(3\sqrt{3}; 1; 1+\sqrt{2})$.

26. 15- rasmda tasvirlangan kub qirralari o‘rtalarining va yoqlari markazlarining koordinatalarini toping.

27. $A(3;-1;4)$, $B(-1;1;-8)$, $C(2;1;-6)$, $D(0;1;2)$ nuqtalar berilgan. a) AB va CD; b) AC va BD kesmalar o‘rtasining koordinatalarini toping.

28. $M(1;-1;2)$ va $N(-3;2;4)$ nuqtalar AB kesmani uchta teng bo‘laklarga ajratadi. AB kesma uchlarining koordinatalarini toping.

29. $ABCD$ to‘rtburchakning tomonlari va $A_1B_1C_1D_1$ to‘g‘ri to‘rtburchakning tomonlariga mos ravishda parallel. $ABCD$ – to‘g‘ri to‘rtburchak ekanini isbotlang?

30. $ABCD$ to‘g‘ri to‘rtburchakning A uchidan uning tekisligiga perpendikular AK to‘g‘ri chiziq o‘tkazilgan. K nuqtadan to‘g‘ri to‘rtburchakning boshqa uchlarigacha bo‘lgan masofalar 6 cm, 7 cm va 9 cm. AK kesmaning uzunligini toping.

31*. Fazoda $A(3; 0; -1)$, $B(-4; 1; 0)$, $C(5; -2; -1)$ nuqtalar berilgan. Oyz tekislikda A, B, C nuqtalardan baravar uzoqlikda joylashgan nuqtani toping.

32. $ABCD$ parallelogrammning uchlari: a) $A(-2; -4; 3)$, $B(3; 1; 7)$, $C(4;2;-5)$; b) $A(4; 2; -1)$, $B(1; -3; -2)$, $C(-6; 2; 1)$; c) $A(-1; 7; 4)$, $B(1; 5; 2)$, $C(9; -3; -8)$ bo‘lsa, D uchining koordinatalarini toping.

33. CK kesmani $CK:KM = \lambda$ nisbatda bo‘luvchi $M(x; y; z)$ nuqtaning koordinatalarini toping. a) $C(-5; 4; 2)$, $K(1; 1;-1)$ va $\lambda=2$; b) $C(1; -1; 2)$, $K(2; -4; 1)$ va $\lambda=0{,}5$; c) $C(1; 0; -2)$, $K(9; -3; 6)$ va $\lambda=\frac{1}{3}$.

34. Uchlari $A(3; 2; 4)$, $B(1; 3; 2)$, $C(-3; 4; 3)$ nuqtalarda bo‘lgan uchburchak medianalari kesishish nuqtasi M ning koordinatalarini toping.

35. Uchlari $A(5; 6; 3)$, $B(3; 5; 1)$, $C(0; 1; 1)$ nuqtalarda bo‘lgan uchburchakning BL bissektrisasining L uchi koordinatalarini toping.

36*. Uchlari $A(4; 0; 1)$, $B(5; -2; 1)$, $C(4; 8; 5)$ nuqtalarda bo‘lgan uchburchakning AL bissektrisasi uzunligini toping.

37*. Uchlari $A(1; 3; -1)$, $B(3; -1; 1)$, $C(3; 1; -1)$ nuqtalar bo‘lgan uchburchak berilgan. Uning: a) katta tomoniga tushirilgan balandligini; b) burchaklarini; c) yuzini toping.

38*. 16- rasmda tasvirlangan kub haqidagi ma’lumotlardan foydalanib MK kesma uzunligini toping.

Tarixiy ma’lumotlar

Abu Rayhon Beruniy mashhur tabib va matematik Abu Ali ibn Sino bilan yozishmalarida unga quyidagi savolni beradi: „Nima uchun Aristotel va boshqa (faylasuf)lar tomonlarni oltita deb atashadi?”

Beruniy olti yoqli kubni olib, „boshqacha sondagi tomonlarga ega bo‘lgan” jismlar haqida gapiradi va „sharsimon jismning tomonlari yo‘qligi”ni qo‘shib qo‘yadi.

Ibn Sino esa „hamma hollarda ham tomonlar oltita deb hisoblamoq zarur, chunki har bir jismda, uning shaklidan qat’iy nazar uch o‘lchov — uzunlik, chuqurlik va kenglik mavjud” deb javob beradi.

Bu yerda Ibn Sino „olti tomon” deb ishoralari bilan olingan uchta koordinatani nazarda tutadi.

Beruniy „Qonuniy Mas’udiy” asarida olti tomonning aniq matematik ma’nosini keltiradi: „Tomonlar oltita, chunki ular jismlarning o‘lchovlari bo‘yicha harakatlari chegarasidir. O‘lchovlar uchta, bu uzunlik, kenglik va chuqurlik, ularning uchlari esa o‘lchovlardan ikki marta ko‘p”.

Asarning oldingi kitoblarida muallif yoritgichlarning osmondagi holatini osmon sferasiga nisbatan ikki koordinata – ekliptik kenglama va uzoqlama orqali yoki xuddi shunday koordinatalar orqali, ammo osmon ekvatori yoki gorizontga nisbatan aniqlaydi. Ammo yulduzlar va yoritgichlarning o‘zaro joylashuvini aniqlash masalasida ularning bir-birlarini to‘sib qolish hollarini ham e’tiborga olishga to‘g‘ri keladi. Mana shunday holda uchinchi sferik koordinataga ehtiyoj tug‘iladi.Ana shu ehtiyoj Abu Rayhon Beruniyni fazoviy koordinatalar g‘oyasini ilgari surishga olib kelgan.

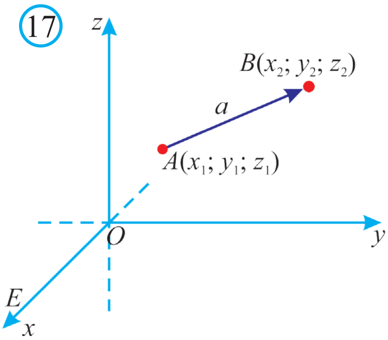
2. FAZODA VEKTORLAR VA ULAR USTIDA AMALLAR

2.1. Fazoda vektorlar

Fazoda vektor tushunchasi tekislikdagi singari kiritiladi.

Fazoda *vektor* deb yo‘naltirilgan kesmaga aytiladi.

Fazoda vektorlarga oid asosiy tushunchalar: vektorning uzunligi (moduli), vektorning yo‘nalishi, vektorlarning tengligi tekislikdagi singari ta’riflanadi.

17


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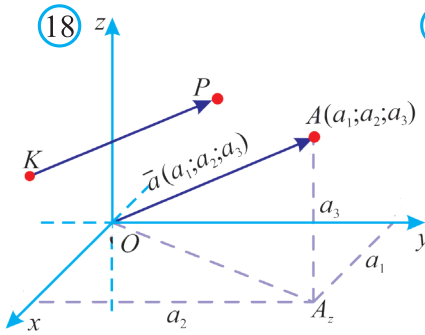

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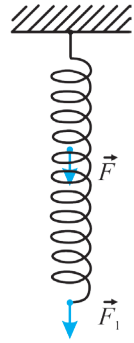


$\overline{a}(a_1;a_2;a_3)$ $\qquad$ $\overline{AB}(x_2-x_1;y_2-y_1;z_2-z_1)$

Boshi $A\ (x_1; y_1; z_1)$ nuqtada va oxiri $B\ (x_2; y_2; z_2)$ nuqtada bo‘lgan *vektorning koordinatalari* deb $a_1 = x_2 - x_1$, $a_2 = y_2 - y_1$, $a_3 = z_2 - z_1$ sonlarga aytiladi (17- rasm).

Vektorlarning tekislikdagiga o‘xshash qator xossalari ham borki, ularni isbotsiz keltiramiz.

Xuddi tekislikdagi singari teng vektorlarning mos koordinatalari teng bo‘ladi va aksincha, mos koordinatalari teng bo‘lgan vektorlar teng bo‘ladi.

Bu vektorni uning koordinatalari bilan ifodalashga asos bo‘ladi. Vektorlar $\overline{AB}(a_1;a_2;a_3)$ yoki $\overline{a}(a_1;a_2;a_3)$ yoki qisqaroq $\overline{(a_1;\ a_2;\ a_3)}$ tarzda belgilanadi (18-rasm).

Vektor koordinatalarisiz $\overline{AB}$ (yoki qisqaroq $\overline{a}$) tarzda ham belgilanadi. Bunda uning boshi birinchi o‘rinda, oxiri esa ikkinchi o‘rinda yoziladi.

Koordinatalari nollardan iborat vektor *nol vektor* deb ataladi va $\overline{0}(0;0;0)$ yoki $\overline{0}$ tarzda belgilanadi hamda bu vektorning yo‘nalishi bo‘lmaydi.

Agar O koordinata boshi va a_1, a_2 va a_3 sonlar A nuqtaning koordinatalari, ya’ni $A(a_1; a_2; a_3)$ bo‘lsa, bu sonlar $\overline{OA}$ vektorning ham koordinatalari bo‘ladi: $\overline{OA}(a_1; a_2; a_3)$.

Lekin koordinatalar fazosida boshi $K(c_1;\ c_2;\ c_3)$ nuqtada, oxiri $P\ (c_1+a_1;\ c_2+a_2;\ c_3+a_3)$ nuqtada bo‘lgan $\overline{KP}$ vektor ham shu koordinatalar bilan ifodalanadi: $\overline{KP}(c_1+a_1-c_1;\ c_2+a_2-c_2;\ c_3+a_3-c_3) = \overline{KP}\ (a_1;\ a_2;\ a_3)$.

Shundan kelib chiqib, vektorni koordinatalar fazosida istalgan nuqtaga qo‘yilgan qilib tasvirlash mumkin. Geometriyada biz shunday *erkin* vektorlar bilan ish ko‘ramiz. Fizikada esa, odatda, vektorlar biror *nuqtaga qo‘yilgan* bo‘ladi. Masalan, 19- rasmdagi F kuch prujinaning qaysi nuqtasiga qo‘yilgani bilan ahamiyatli hisoblanadi.

Vektorning uzunligi deb uni tasvirlovchi yo‘naltirilgan kesmaning uzunligiga aytiladi (17- rasm). $\bar{a}$ vektorning uzunligi $|\bar{a}|$ tarzda ifodalanadi.

$\bar{a}(a_1;\ a_2;\ a_3)$ vektorning uzunligi uning koordinatalari orqali $|\bar{a}|=\sqrt{a_1^{\ 2}+a_2^{\ 2}+a_3^{\ 2}}$ formula bilan ifodalanadi.

1- masala. $A(2;\ 7;\ –3)$, $B(1;\ 0;\ 3)$, $C(–3;\ –4;\ 5)$ va $D(–2;\ 3;\ –1)$ nuqtalar berilgan. $\overline{AB}$, $\overline{BC}$, $\overline{DC}$, $\overline{AD}$, $\overline{AD}$ va $\overline{BD}$ vektorlardan qaysilari o‘zaro teng bo‘ladi?

Yechish: Teng vektorlarning mos koordinatalari teng bo‘ladi. Shuning uchun vektorlarning koordinatalarini topamiz:

$\overline{AB} = (1-2,\ 0-7,\ 3-(-3)) = (-1,\ -7,\ 6);$

$\overline{DC} = (-3-(-2),\ -4-3,\ 5-(-1)) = (-1,\ -7,\ 6).$

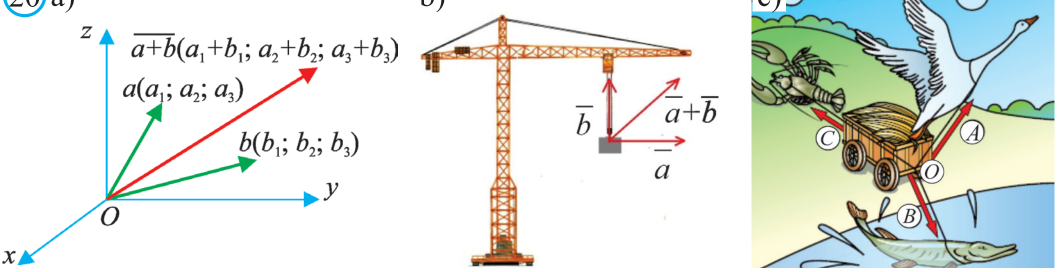
Demak, $\overline{AB} = \overline{DC}$. $\overline{BC}=\overline{AD}$ ekanligini mustaqil ko‘rsating. □

2.2. Fazoda vektorlar ustida amallar

Vektorlar ustida amallar. Vektorlarni qo‘shish, songa ko‘paytirish va skalar ko‘paytirish amallari xuddi tekislikdagidek aniqlanadi.

$\bar{a}(a_1;\ a_2;\ a_3)$ va $\bar{b}(b_1;\ b_2;\ b_3)$ *vektorlarning yig‘indisi* deb $\bar{a}+\bar{b}=(a_1+b_1;\ a_2+b_2;\ a_3+b_3)$ vektorga aytiladi (20-rasm).

20 a) b) c)



20.*b*-rasmda kran $\bar{a}$ vektor bo‘yicha, yuk esa kranga nisbatan $\bar{b}$ vektor bo‘yicha harakatlanayotgan bo‘lsin. Natijada yuk $\bar{a}+\bar{b}$ vektor bo‘yicha harakatlanadi. Shuningdek, 20.*c*- rasmda tasvirlangan rus yozuvchisi Krilovning masali qahramonlari nima sababdan aravani joyidan qo‘zg‘ata olmayotganini sezgan bo‘lsangiz kerak.

Vektorlar yig‘indisining xossalari.

Ixtiyoriy $\bar{a}$, $\bar{b}$ va $\bar{c}$ vektorlar uchun quyidagi xossalar o‘rinli:

a) $\bar{a}+\bar{b}=\bar{b}+\bar{a}$ – vektorlarni qo‘shishning o‘rin almashtirish qonuni;

b) $\bar{a}+(\bar{b}+\bar{c})=(\bar{a}+\bar{b})+\bar{c}$ – vektorlarni qo‘shishning taqsimot qonuni.

Vektorlarni qo‘shishning uchburchak qoidasi.

Ixtiyoriy *A*, *B* va *C* nuqtalar uchun (21-rasm): $\overline{AB}+\overline{BC}=\overline{AC}$.

Vektorlarni qo‘shishning parallelogramm qoidasi.

Agar *ABCD* – parallelogramm (22- rasm) bo‘lsa, $\overline{AB}+\overline{AD}=\overline{AC}$.

Vektorlarni qo‘shishning ko‘pburchak qoidasi.

Agar *A, B, C, D* va *E* nuqtalar ko‘pburchak uchlari bo‘lsa (23- rasm),

$$\overline{AB}+\overline{BC}+\overline{CD}+\overline{DE}=\overline{AE} \quad \text{bo‘ladi.}$$

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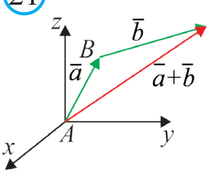

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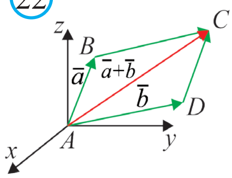

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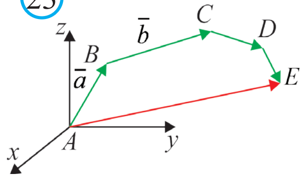

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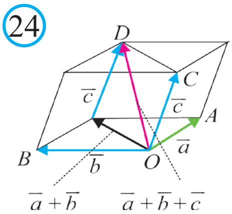

Bir tekislikda yotmagan uchta *vektorlarni qo‘shishning parallelepiped* qoidasi. Agar $ABCDA_1B_1C_1D_1$ parallelepiped (24- rasm) bo‘lsa,

$$\overline{AB}+\overline{AD}+\overline{AA_1}=\overline{AC} \quad \text{bo‘ladi.}$$

$\bar{a}(a_1;a_2;a_3)$ *vektorning* λ *songa ko‘paytmasi* deb $\lambda\bar{a}=(\lambda a_1;\lambda a_2;\lambda a_3)$ vektorga aytiladi (25- rasm).

Ixtiyoriy $\bar{a}$ va $\bar{b}$ vektorlar hamda λ va μ sonlar uchun

a) $\lambda(\bar{a}+\bar{b})=\lambda\bar{a}+\lambda\bar{b}$;

b) $(\lambda+\mu)\bar{a}=\lambda\bar{a}+\mu\bar{a}$;

c) $|\lambda\bar{a}|=|\lambda|\cdot|\bar{a}|$ va $\lambda\bar{a}$ vektorning yo‘nalishi

$\lambda>0$ bo‘lganda, $\bar{a}$ vektor yo‘nalishi bilan bir xil va

$\lambda<0$ bo‘lganda, $\bar{a}$ vektor yo‘nalishiga qarama-qarshi bo‘ladi.

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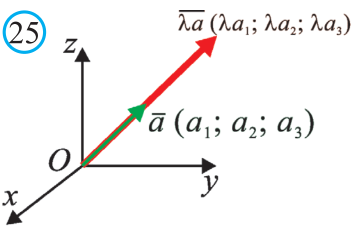
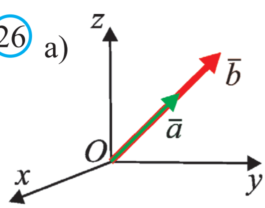


26 a)


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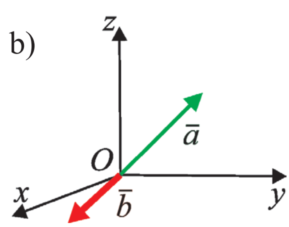


2.3. Kollinear va komplanar vektorlar

Nol vektordan farqli $\overline{a}$ va $\overline{b}$ vektorlar berilgan bo‘lsin. $\overline{a}$ va $\overline{b}$ vektorlar bir xil yoki qarama-qarshi yo‘nalgan bo‘lsa, ular *kollinear vektorlar* deb ataladi (26- rasm).

1- xossa. $\overline{a}$ va $\overline{b}$ vektorlar uchun $\overline{a} = \lambda\overline{b}\,(\lambda \neq 0)$ tenglik o‘rinli bo‘lsa, ular o‘zaro kollinear bo‘ladi va aksincha.

Agar $\lambda > 0$ bo‘lsa, $\overline{a}$ va $\overline{b}$ vektorlar bir tomonga ($\overline{a} \uparrow\uparrow \overline{b}$), agar $\lambda < 0$ bo‘lsa, qarama-qarshi tomonga ($\overline{a} \uparrow\downarrow \overline{b}$) yo‘nalgan bo‘ladi.

2- xossa. $\overline{a}(a_1;\, a_2;\, a_3)$ va $\overline{b}(b_1;\, b_2;\, b_3)$ vektorlar o‘zaro kollinear bo‘lsa, ularning koordinatalari o‘zaro proporsional bo‘ladi: $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ va aksincha.

2- masala. Boshi $A\,(1;\,1;\,1)$ nuqtada va oxiri Oxy tekislikdagi B nuqtada bo‘lgan va $\overline{a}(1;\,2;\,3)$ vektorga kollinear vektorni toping.

Yechish: B nuqtaning koordinatalari $B(x;\, y;\, z)$ bo‘lsin. B nuqta Oxy tekislikda yotgani uchun $z=0$. Unda $\overline{AB}\,(x-1;\, y-1;\, -1)$ bo‘ladi.

Shartga ko‘ra, $\overline{AB}\,(x-1;\, y-1;\, -1)$ va $\overline{a}(1,\, 2,\, 3)$ vektorlar kollinear. Demak, ularning koordinatalari o‘zaro proportsional bo‘ladi.

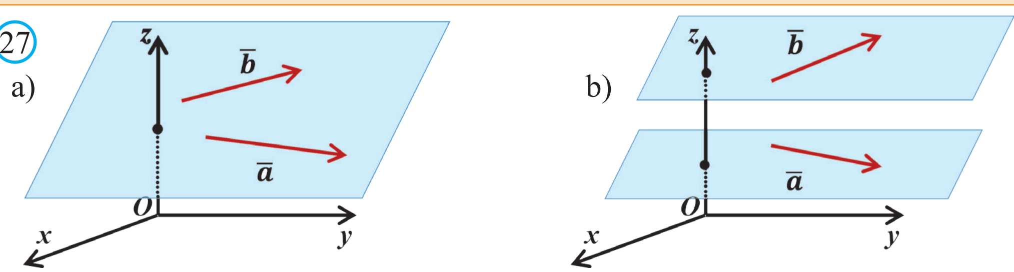
Bundan $\frac{x-1}{1} = \frac{y-1}{2} = \frac{-1}{3}$ proporsiyalarni hosil qilamiz.

Ulardan $x = \frac{2}{3}$, $y = \frac{1}{3}$ ekanligini topamiz.

Unda $\overline{AB}\left(-\frac{1}{3}; -\frac{2}{3}; -1\right)$ bo‘ladi. □

Bitta tekislikda yoki parallel tekisliklarda yotuvchi vektorlar *komplanar vektorlar* deb ataladi (27- rasm).

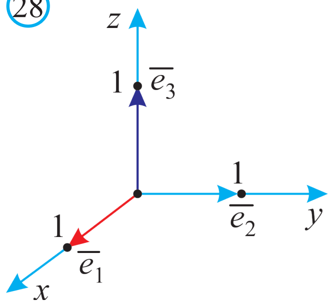
27



$\overline{e_1}\,(1;\,0;\,0)$, $\overline{e_2}\,(0;\,1;\,0)$ va $\overline{e_3}\,(0;\,0;\,1)$ vektorlar *ortlar* deb ataladi (28- rasm).

Ixtiyoriy $\overline{a}\,(a_1;\, a_2;\, a_3)$ vektorni $\overline{a} = a_1\overline{e_1} + a_2\overline{e_2} + a_3\overline{e_3}$ ko‘rinishda, yagona tarzda *ortlar bo‘yicha yoyish* mumkin (29- rasm).

28



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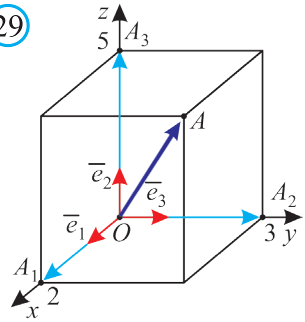


Shuningdek, uchta komplanar bo'lmagan $\overline{OA}, \overline{OB}$ va $\overline{OC}$ vektorlar berilgan bo'lsa, ixtiyoriy $\overline{OD}$ vektorni quyidagi ko'rinishda, yagona tarzda ifodalash mumkin:

$$\overline{OD} = a_1 \cdot \overline{OA} + a_2 \cdot \overline{OB} + a_3 \cdot \overline{OC}.$$

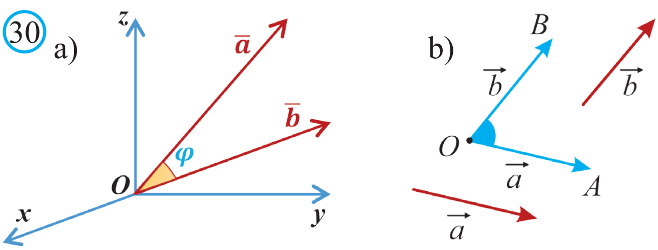
Bu yerda a_1, a_2, a_3 qandaydir haqiqiy sonlar. Bunga *vektorni berilgan vektorlar bo'yicha yoyish* deb ataladi.

2.4. Vektorlarning skalar ko'paytmasi

Nol vektordan farqli $\overline{a}$ va $\overline{b}$ *vektorlar orasidagi burchak* deb O nuqtadan chiquvchi $\overline{OA} = \overline{a}$ va $\overline{OB} = \overline{b}$ vektorlarning yo'naltiruvchi kesmalari orasidagi burchakka aytiladi (30- rasm).

$\overline{a}$ va $\overline{b}$ vektorlar orasidagi burchak $(\widehat{\overline{a}, \overline{b}})$ tarzda ham belgilanadi.

30



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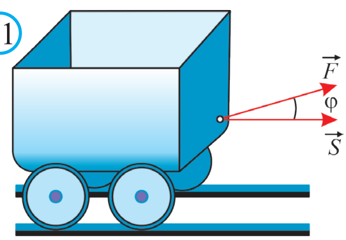


$\overline{a}$ va $\overline{b}$ *vektorlarning skalar ko'paytmasi* deb, bu vektorlar uzunliklarining ular orasidagi burchak kosinusi ko'paytmasiga aytiladi.

Agar vektorlarning biri nol vektor bo'lsa, ularning skalar ko'paytmasi nolga teng bo'ladi.

Skalar ko'paytma $\overline{a} \cdot \overline{b}$ yoki $(\overline{a}; \overline{b})$ tarzda belgilanadi. Ta'rifga ko'ra

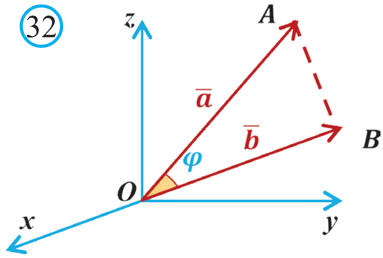
$$(\overline{a}; \overline{b}) = |\overline{a}| \cdot |\overline{b}| \cos\varphi. \qquad (1)$$

Ta'rifdan ko'rinadiki, $\overline{a}$ va $\overline{b}$ vektorlarning skalar ko'paytmasi nolga teng bo'lsa, ular *perpendikular* bo'ladi va aksincha.

Fizikada jismni $\overline{F}$ kuch ta'siri ostida $\overline{s}$ masofaga siljitishda bajarilgan A ish (31- rasm) $\overline{F}$ va $\overline{s}$ vektorlarning skalar ko'paytmasiga teng bo'ladi:

$$A = (\overline{F}, \overline{s}) = |\overline{F}| \cdot |\overline{s}| \cos\varphi.$$

Xossa. $\bar{a}(a_1; a_2; a_3)$ va $b(b_1; b_2; b_3)$ vektorlar uchun $(\bar{a}; \bar{b}) = a_1b_1 + a_2b_2 + a_3b_3$.



Isbot. $\bar{a}$ va $\bar{b}$ vektorlarni koordinata boshi O nuqtaga qo'yamiz (32- rasm). Unda $\overline{OA} = (a_1; a_2; a_3)$ va $\overline{OB} = (b_1; b_2; b_3)$ bo'ladi. Agar berilgan vektorlar kollinear bo'lmasa, ABO uchburchakdan iborat bo'ladi va uning uchun kosinuslar teoremasi o'rinli bo'ladi:

$$AB^2 = OA^2 + OB^2 - 2OA \cdot OB \cdot \cos\varphi.$$ Unda

$OA \cdot OB \cdot \cos\varphi = \frac{1}{2}(OA^2 + OB^2 - AB^2)$ bo'ladi. Lekin, $OA^2 = a_1^2 + a_2^2 + a_3^2$,

$OB^2 = b_1^2 + b_2^2 + b_3^2$ va $AB^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2$.

Demak, $(\bar{a}, \bar{b}) = |\bar{a}| \cdot |\bar{b}| \cos\varphi = \frac{1}{2}(OA^2 + OB^2 - AB^2) =$

$$= \frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 - (b_1 - a_1)^2 - (b_2 - a_2)^2 -$$

$$-(b_3 - a_3)^2) = a_1b_1 + a_2b_2 + a_3b_3.$$

Berilgan vektorlar kollinear bo'lgan ($\varphi = 0°$, $\varphi = 180°$) holda ham bu tenglik o'rinli bo'lishini mustaqil ko'rsating. □

Vektorlarning skalar ko'paytmasining xossalari

1. $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ – o'rin almashtirish xossasi.

2. $(\bar{a} + \bar{b}) \cdot \bar{c} = \bar{a} \cdot \bar{c} + \bar{b} \cdot \bar{c}$ – taqsimot xossasi.

3. $\lambda \cdot (\bar{a} \cdot \bar{b}) = (\lambda \cdot \bar{a}) \cdot \bar{b} = \bar{a} \cdot (\lambda \cdot \bar{b})$ – guruhlash xossasi.

4. Agar a va b vektorlar bir xil yo'nalishdagi kollinear vektorlar bo'lsa, $\bar{a} \cdot \bar{b} = |\bar{a}|\,|\bar{b}|$ bo'ladi, chunki $\cos 0° = 1$.

5. Agar qarama-qarshi yo'nalgan bo'lsa, $\bar{a} \cdot \bar{b} = -|\bar{a}|\,|\bar{b}|$, chunki $\cos 180° = -1$.

6. $\bar{a} \cdot \bar{a} = |\bar{a}||\bar{a}| \cos 0° = |\bar{a}|^2 \Rightarrow \bar{a}^2 = |\bar{a}|^2$.

7. $\bar{a}$ vektor $\bar{b}$ vektorga perpendikular bo'lsa, $\bar{a} \cdot \bar{b} = 0$ bo'ladi.

Natijalar:

a) $\bar{a} = (a_1; a_2; a_3)$ vektorning uzunligi: $|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; (1)

b) $\bar{a} = (a_1; a_2; a_3)$ va $\bar{b} = (b_1; b_2; b_3)$ vektorlar orasidagi burchak kosinusi:

$$\cos\varphi = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}; \quad (2)$$

c) $\overline{a}=(a_1;a_2;a_3)$ va $\overline{b}=(b_1;b_2;b_3)$ vektorlarning perpendikularlik sharti:
$$a_1b_1+a_2b_2+a_3b_3=0. \qquad (3)$$

3- masala. $A(0;\ 1;\ -1)$, $B(1;\ -1;\ 2)$, $C(3;\ 1;\ 0)$, $D(2;\ -3;\ 1)$ nuqtalar berilgan. $\overline{AB}$ va $\overline{CD}$ vektorlar orasidagi burchakning kosinusini toping.

Yechish. $\overline{AB}$ va $\overline{CD}$ vektorlarning koordinatalarini so'ng uzunliklarini topamiz: $\overline{AB} = (1-0;\ -1-1;\ 2-(-1)) = (1, -2, 3)$,
$\overline{CD} = (2-3;\ -3-1;\ 1-0) = (-1, -4, 1)$.
$$|\overline{AB}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{14},$$
$$|\overline{CD}|=\sqrt{(-1)^2+(-4)^2+1^2}=\sqrt{18}.$$
Demak, $\cos\varphi=\dfrac{\overline{AB}\cdot\overline{CD}}{|\overline{AB}|\cdot|\overline{CD}|}=\dfrac{1\cdot(-1)+(-2)(-4)+3\cdot1}{\sqrt{14}\cdot\sqrt{18}}=\dfrac{5}{\sqrt{63}}$. □

4- masala. $\overline{a}(1;2;0)$, $\overline{b}(1;-\frac{1}{2};0)$ vektorlar orasidagi burchakni toping.

Yechish: $\cos\varphi=\dfrac{\overline{a}\cdot\overline{b}}{|\overline{a}|\cdot|\overline{b}|}=\dfrac{1\cdot1+2\left(-\frac{1}{2}\right)+0\cdot0}{\sqrt{1^2+2^2+0^2}\sqrt{1^2+\left(-\frac{1}{2}\right)^2+0^2}}=\dfrac{0}{\sqrt{5}\sqrt{\frac{5}{4}}}=0$.

Demak, $\varphi=90°$. □

5- masala. $|\overline{a}|=3$, $|\overline{b}|=5$ va bu vektorlar orasidagi burchak $\dfrac{2\pi}{3}$ ga teng bo'lsa, $|\overline{a}+\overline{b}|$ ni toping.

Yechish: $|\overline{a}+\overline{b}|=\sqrt{(\overline{a}+\overline{b})^2}=\sqrt{\overline{a}^2+2(\overline{a}\cdot\overline{b})+\overline{b}^2}=\sqrt{|\overline{a}|^2+2|\overline{a}||\overline{b}|\cos\phi+|\overline{b}|^2}=$
$$=\sqrt{9+25+2\cdot15\cdot\left(-\frac{1}{2}\right)}=\sqrt{34-15}=\sqrt{19}$$

6- masala. Agar $\overline{a}=2\overline{i}+3\overline{j}-4\overline{k}$ va $\overline{b}=-\overline{i}-\overline{j}+2\overline{k}$ bo'lsa, 1) $\overline{c}=\overline{a}+\overline{b}$; 2) $\overline{d}=2\overline{a}-\overline{b}$ vektorning koordinatalarini va uzunligini toping.

Yechish: $\overline{a}$ va $\overline{b}$ vektorlar yoyilmalarini koordinatalari izlanayotgan vektor ifodasiga qo'yamiz: 1) $\overline{c}=\overline{a}+\overline{b}=2\overline{i}+3\overline{j}-4\overline{k}-\overline{i}-\overline{j}+2\overline{k}=\overline{i}+2\overline{j}-2\overline{k}$.

Demak, $\overline{c}=\overline{(1;2;-2)}$. Unda $|\overline{c}|=\sqrt{1^2+2^2+(-2)^2}=3$;

2) $\overline{d}=2\overline{a}-\overline{b}=2(2\overline{i}+3\overline{j}-4\overline{k})-(-\overline{i}-\overline{j}+2\overline{k})=4\overline{i}+6\overline{j}-8\overline{k}+\overline{i}+\overline{j}2\overline{k}=$
$=5\overline{i}+7\overline{j}-10\overline{k}$.

Demak, $\overline{d}=\overline{(5;7;-10)}$. Unda $|\overline{d}|=\sqrt{5^2+7^2(-10)^2}=\sqrt{174}$. □

7- masala. $\overline{a}$ va $\overline{b}$ vektorlar orasidagi burchak 30° ga teng va $|\overline{a}| = \sqrt{3}$, $|\overline{b}|=2$ bo‘lsa, $(2\overline{a} + 3\overline{b})(-2\overline{a}+\overline{b})$ ko‘paytmani hisoblang.

Yechish: Avval $\overline{a}$ va $\overline{b}$ vektorlar ko‘paytmasini hisoblaymiz:

$$(\overline{a}, \overline{b}) = |\overline{a}||\overline{b}| \cos 30° = \sqrt{3}\cdot 2\cdot\frac{\sqrt{3}}{2} = 3.$$

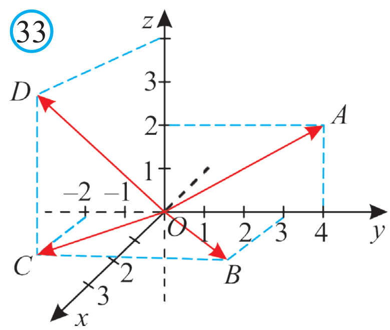
So‘ng vektorlar ko‘paytmasining taqsimot xossasiga ko‘ra, berilgan vektorlar ifodalarini ko‘phadni ko‘phadga ko‘paytirish kabi ko‘paytiramiz:

$$(2\overline{a}+3\overline{b})(-2\overline{a}+\overline{b})=-4\overline{a}^2 +2(\overline{a}, \overline{b})-6(\overline{a}, \overline{b})+3\overline{b}^2=-4\overline{b}^2-4(\overline{a}, \overline{b})+3\overline{b}^2.$$

$\overline{a}^2=|\overline{a}|^2=9$, $\overline{b}=|\overline{b}|^2=4$, $(\overline{a}, \overline{b})=3$ ekanligini hisobga olsak, izlanayotgan ko‘paytma $(2\overline{a}+3\overline{b})(-2\overline{a}+\overline{b})=-4\cdot 9-4\cdot 3+3\cdot 4=-36.$ □

Mavzuga oid masalalar va amaliy topshiriqlar

39. 33- rasmdagi vektorlarning koordinatalarini aniqlang.



40. $A(1; 1; 1)$, $B(-1; 0; 1)$, $C(0; 1; 1)$ va $O(0; 0; 0)$ nuqtalar berilgan.

$\overline{OA}, \overline{OB}, \overline{OC}, \overline{BO}, \overline{CO}$ va $\overline{AB}$ vektorlar koordinatalarini aniqlang.

41. $\overline{AB}$ $(a; b; c)$ bo‘lsa, $\overline{BA}$ vektor koordinatalarini ayting.

42. Agar a) $A(1; 2; 3)$, $B(3; 7; 6)$; b) $A(-3; 2; 1)$, $B(1; -4; 3)$ bo‘lsa, $\overline{AB}$ vektor koordinatalarini toping.

43. $\overline{a}(1; -1; 1)$, $\overline{b}(0; 2; -4)$, $\overline{c}(2; 3; -1)$, $\overline{d}(1; 2; 5)$ vektorlarning uzunligini toping.

44. Agar $\overline{a}(2; 1; 3)$ va $\overline{b}(-1; x; 2)$ vektorlar uzunligi teng bo‘lsa, x ni toping.

45. Uzunligi $\sqrt{54}$ ga teng bo‘lgan $\overline{a}(c; 2c; -c)$ vektorning koordinatalarini toping.

46. A, B, C, D, E va F nuqtalar muntazam oltiburchakning uchlari bo‘lsa, ular orqali: a) ikkita teng; b) ikkita bir xil yo‘nalgan; c) ikkita qarama-qarshi yo‘nalgan va teng; d) ikkita qarama-qarshi yo‘nalgan va teng bo‘lmagan vektorlarga misol keltiring.

47. k ning qanday qiymatida: a) $\overline{a}(4; k; 2)$; b) $\overline{a}(k-1;1;4)$; c) $\overline{a}(k; 1; k+2)$; d) $\overline{a}(k-1; k-2; k+1)$ vektorning uzunligi $\sqrt{21}$ ga teng bo‘ladi?

48. Uchta nuqta berilgan: $A(1; 1; 1)$, $B(-1; 0; 1)$, $C(0; 1; 1)$. Shunday

$D(x; y; z)$ nuqtani topingki, $\overline{AB}$ va $\overline{CD}$ vektorlar teng bo'lsin.

49. Uchta nuqta berilgan: $A(1; 0; 1)$, $B(-1; 1; 2)$, $C(0; 2; -1)$. Agar a) $\overline{AB}$ va $\overline{CD}$ vektorlar teng; b) $\overline{AB}$ va $\overline{CD}$ vektorlarning yig'indisi nol vektorga teng bo'lsa, $D(x; y; z)$ nuqtani toping.

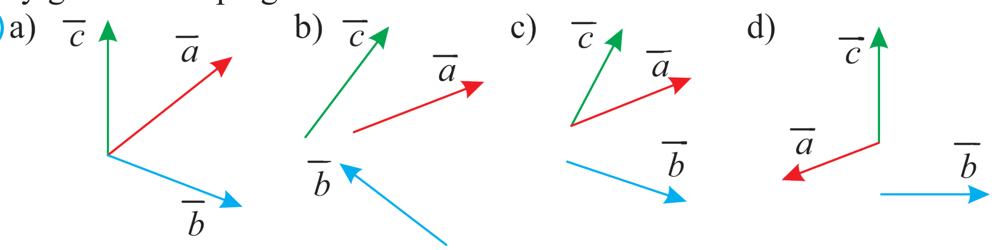
50*. $(2; n; 3)$ va $(3; 2; m)$ vektorlar berilgan. m va n ning qanday qiymatlarida bu vektorlar kollinear bo'ladi?

51. Boshi $A(1; 1; 1)$ nuqtada va oxiri Oxy tekislikdagi B nuqtada bo'lgan hamda $a(1;-2; 3)$ vektorga kollinear vektorni toping.

52*. $ABCD$ parallelogrammning uchlari a) $A(-2; -4; 3)$, $B(3; 1; 7)$, $C(4; 2; -5)$; b) $A(4; 2; -1)$, $B(1; -3; -2)$, $C(-6; 2; 1)$; c) $A(-1; 7; 4)$, $B(1; 5; 2)$, $C(9; -3; -8)$; d) $A(-2; -4; 3)$, $B(3; 1; 7)$, $C(4; 2; -5)$ bo'lsa, D uchining koordinatalarini toping.

53. 34- rasmda tasvirlangan vektorlarning parallelepiped qoidasiga ko'ra yig'indisini toping.

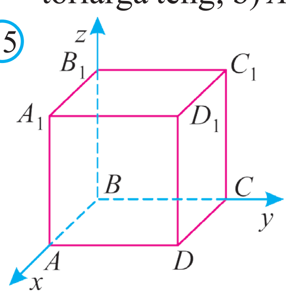
34



54. Agar $A(6;7;8)$, $B(8;2;6)$, $C(4;3;2)$, $D(2;8;4)$ va $M(3;5;2)$, $N(7;1;2)$, $P(3;-3;2)$, $K(-1;1;2)$ bo'lsa, $ABCD$ va $MNPK$ to'rtburchaklardan qaysi biri romb, qaysinisi kvadrat bo'ladi?

55. 35- rasmda tasvirlangan $ABCDA_1B_1C_1D_1$ kubda: a) $\overline{AB}$, $\overline{DD_1}$, $\overline{AC}$ vektorlarga teng; b) $\overline{A_1D_1}$, $\overline{CC_1}$, $\overline{BD}$ vektorlarga qarama-qarshi yo'nalgan; c) $\overline{BA}$, $\overline{AA_1}$ vektorlarga kollinear; d) $\overline{AB}$ va $\overline{AD}$, $\overline{AC}$ va $\overline{A_1C}$ vektorlar juftiga komplanar vektorlarni aniqlang.

35



56. Agar 1) $\overline{a}(1;-4;0)$, $\overline{b}(-4;0;8)$; 2) $\overline{a}(0;2;5)$, $\overline{b}(4; 3; 0)$ bo'lsa, $\overline{c} = \overline{a} + \overline{b}$ vektorning koordinatalarini va uzunligini toping.

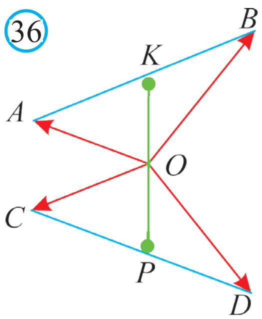
57. Agar 1) $\overline{a}(1; -4; 0)$, $\overline{b}(-4; 8; 0)$; 2) $\overline{a}(0; -2; 7)$, $\overline{b}(0; 4; -1)$ bo'lsa, $\overline{c} = \overline{a} - \overline{b}$ vektorning koordinatalarini va uzunligini toping.

58. Agar $\overline{b}(-4; 8; 2)$ bo'lsa, a) $2\overline{b}$; b) $-3\,\overline{b}$; c) $-1{,}5\,\overline{c}$; d) $0 \cdot \overline{b}$ vektorning koordinatalarini va uzunligini toping.

59. $\overline{a}(1;\ -1;\ 1)$, $\overline{b}(0;\ 2;\ -4)$, $\overline{c}(2;\ 3;\ -1)$, $\overline{d}(1;\ 2;\ 5)$ vektorlarni ortlar bo‘yicha yoying.

60*. $\overline{a}(1;\ -1;\ 1)$, $\overline{b}\ (0;\ 2;\ -4)$, $\overline{c}(2;\ 3;\ -1)$, $\overline{d}(1;\ 2;\ 5)$ vektorlar berilgan. $|\overline{a}+2\overline{b}|$, $|\overline{a}-3\overline{b}|$, $|\overline{c}-2\overline{d}|$, $|3\overline{a}+4\overline{d}|$ ni toping.

61*. *K* va *P* nuqtalar ayqash to‘g‘ri chiziqlarda yotuvchi *AB* va *CD* kesmalarning o‘rtasi hamda *O* nuqta *KP* kesmaning o‘rtasi bo‘lsa (36-rasm), $\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}=\overline{0}$ ekanligini isbotlang.

36


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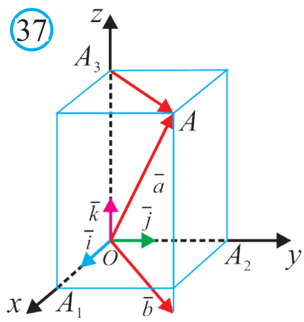

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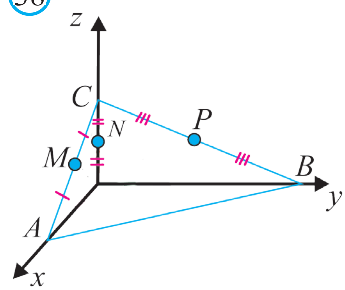
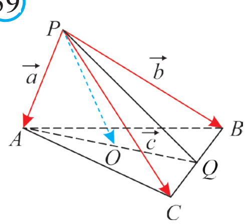


62. 37- rasmda $OA_1=2$, $OA_2=2$, $OA_3=3$. $\overline{a}$, $\overline{b}$ va $\overline{A_3A}$ vektorlarning koordinatalarini aniqlang.

63. 38- rasmda $OA=4$, $OB=9$, $OC=2$, *M*, *N* va *P* nuqtalar, mos ravishda, *AC*, *OC* va *CB* kesmalarning o‘rtasi. $\overline{AC}$, $\overline{CB}$, $\overline{AB}$, $\overline{PC}$, $\overline{MC}$ va $\overline{CN}$ vektorlarning koordinatalarini toping.

64. *Q* nuqta *PABC* tetraedrning *BC* qirrasining o‘rtasi va *O* nuqta esa *AQ* kesma o‘rtasi bo‘lsa (39- rasm), $\overline{PO}$ vektorni $\overline{PA}=\overline{a}$, $\overline{PB}=\overline{b}$ va $\overline{PC}=\overline{c}$ vektorlar orqali ifodalang.

65*. 40- rasmda tasvirlangan qayiqqa daryo oqimi $\overline{F_1}=120\ N$ kuch bilan va qirg‘oqdan esayotgan shamol $\overline{F_2}=100\ N$ kuch bilan ta’sir qilmoqda. Qayiqning daryoda joyidan qo‘zg‘almay turishi uchun uni qanday kuch bilan ushlab turish kerak?

39


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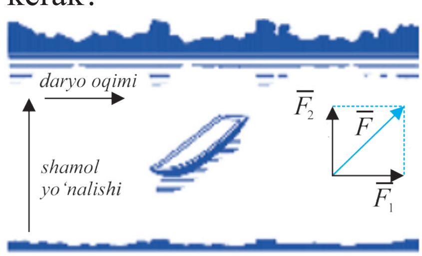

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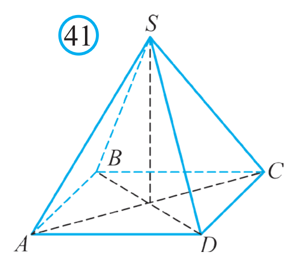


66. Skalar ko‘paytmasi: a) $\frac{1}{2}$; b) $\frac{\sqrt{3}}{2}$; c) 0; d) $-\frac{1}{2}$; e) b) $-\frac{\sqrt{2}}{2}$ ga teng bo‘lgan birlik vektorlar orasidagi burchakni toping.

67. a) $\overline{a}$ (1; –1; 1), $\overline{b}$ (0; 2; –4); b) $\overline{c}$(2; 3; –1), $\overline{d}$ (1; 2; 5); c) $\overline{e}$(1; –1; 1), $\overline{f}$(0; 2; – 4); d) $\overline{g}$(2; 3; –1), $\overline{h}$(1; 2; 5) vektorlarning skalar ko'paytmasini toping.

68. ABC uchburchakda $\angle A=50°$, $\angle C=90°$. a) $\overline{BA}$ va $\overline{BC}$; b) $\overline{CA}$ va $\overline{AB}$; c) $\overline{AB}$ va $\overline{BA}$ vektorlar orasidagi burchakni toping.

69. $\overline{a}$ va $\overline{b}$ vektorlarning uzunliklari va ular orasidagi burchak mos ravishda a) 5, 12, 50°; b) 3, $\sqrt{2}$, 45°; c) 5, 6, 120°; d) 4, 7, 180° bo'lsa, ularning skalar ko'paytmasini toping.

70. n ning qanday qiymatida vektorlar perpendikular bo'ladi?
a) $\overline{a}(2; –1; 3)$, $\overline{b}(1; 3; n)$; b) $\overline{a}(n; –2; 1)$, $\overline{b}(n; –n; 1)$;
c) $\overline{a}(n; –2; 1)$, $\overline{b}(n; 2n; 4)$; d) $\overline{a}(4; 2n; –1)$, $\overline{b}(–1; 1; n)$.

71. $\overline{a}(1;–5;2)$, $\overline{b}(3;1;2)$ vektorlar berilgan. a) $\overline{a}+\overline{b}$ va $\overline{a}-\overline{b}$; b) $\overline{a}+2\overline{b}$ va $3\overline{a}-\overline{b}$; c) $2\overline{a}+\overline{b}$ va $3\overline{a}-2\overline{b}$ vektorlar skalar ko'paytmasini toping.

72. A (1; 0; 1), B (–1; 1; 2), C (0; 2; –1) nuqtalar berilgan. Oz koordinatalar o'qida shunday D nuqtani topingki, $\overline{AB}$ va $\overline{CD}$ vektorlar perpendikular bo'lsin.

73*. $(\overline{a}, \overline{b}) \le |\overline{a}| \cdot |\overline{b}|$ ekanligini asoslang. Bu vektorlar qanday bo'lganda tenglik o'rinli bo'ladi?

74*. $SABCD$ piramidaning hamma qirralari o'zaro teng (41- rasm) va asosi kvadratdan iborat. a) $\overline{SA}$ va $\overline{SB}$; b) $\overline{SD}$ va $\overline{AD}$; c) $\overline{SB}$ va $\overline{SD}$; d) $\overline{AS}$ va $\overline{AC}$; e) $\overline{AC}$ va $\overline{AD}$ vektorlar orasidagi burchaklarni toping

75*. Uzunliklari birga teng $\overline{a}$, $\overline{b}$, $\overline{c}$ vektorlar juft-jufti bilan 60° li burchak tashkil etadi. a) $\overline{a}$ va $\overline{b}+\overline{a}$; b) $\overline{a}$ va $\overline{b}-\overline{c}$ vektorlar orasidagi burchakni toping.

76. O nuqta $ABCD$ kvadratning diagonallari kesishish nuqtasi. Kvadratning B uchidan diagonalga parallel va DA to'g'ri chiziq bilan F nuqtada kesishadigan to'g'ri chiziq o'tkazilgan. $\overline{BF}$ vektorni $\overline{DO}$ va $\overline{DC}$ vektorlar orqali ifodalang.

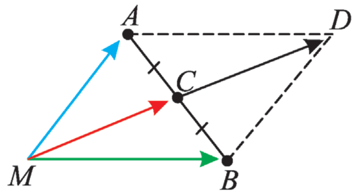
77. O nuqta ABC uchburchakning medianalari kesishish nuqtasi bo'lsa, $\overline{OC}$ vektorni $\overline{AB}$ va $\overline{AC}$ vektorlar bo'yicha yoying.

78*. C nuqta AB kesmaning o'rtasi bo'lsa (42- rasm), unda ixtiyoriy M nuqta uchun $\overline{MC}=\frac{1}{2}(\overline{MA}+\overline{MB})$ bo'lishini isbotlang.

79. K nuqta $ABCD$ tetraedr BC qirrasining o'rtasi bo'lsa (43- rasm), $\overline{DK}$ vektorni $\overline{AB}$, $\overline{AD}$ va $\overline{AC}$ vektorlar bo'yicha yoying.

80*. Jismning siljish yo'nalishiga nisbatan 30° li burchak ostida qo'yilgan $\overline{F}=20N$ kuch ta'sirida jism 3 m ga siljidi. Bu holatda bajarilgan ishni toping.

42



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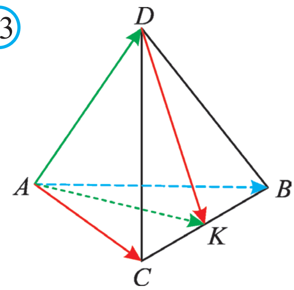


81*. Jismning siljish yo'nalishiga nisbatan 60° li burchak ostida qo'yilgan $\overline{F}$ =50 N kuch ta'sirida jism 8 m ga siljidi. Bu holatda bajarilgan ishni toping.

82*. (Koshi – Bunyakovskiy tengsizligi) Ixtiyoriy $a_1, a_2, a_3, b_1, b_2, b_3$ sonlari uchun $(a_1b_1 + a_2b_2 + a_3b_3)^2 \leq (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$ tengsizlikning o'rinli bo'lishini vektorlardan foydalanib isbotlang.

3. FAZODA ALMASHTIRISHLAR VA O'XSHASHLIK

3.1. Fazoda geometrik almashtirishlar

Fazoda berilgan F shaklning har bir nuqtasi biror bir usulda ko'chirilsa, yangi F_1 shakl hosil bo'ladi. Agar bu ko'chirishda (akslantirishda) birinchi shaklning har xil nuqtalari ikkinchi shaklning har xil nuqtalariga ko'chsa, bu ko'chishga *geometrik shakl almashtirish* deb ataladi.

Butun fazoni ham geometrik shakl sifatida qarasak, fazoviy shakl almashtirish haqida ham gapirish mumkin.

Ko'rib turganingizdek, fazoda geometrik almashtirishlar tushunchasi tekislikdagi kabi kiritiladi. Shuningdek, uning quyida ko'riladigan qator turlarining xossalari va ularning isboti ham tekislikdagisiga o'xshash. Shu bois, bu xossalarning isbotiga to'xtalmaymiz va ularni mustaqil bajarishni tavsiya qilamiz.

3.2. Harakat va parallel ko'chish

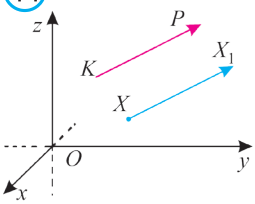
Nuqtalar orasidagi masofani saqlaydigan shakl almashtirishlar *harakat* deb ataladi. Harakatning quyidagi xossalarini keltirish mumkin.

Harakatda to'g'ri chiziq to'g'ri chiziqqa, nur-nurga, kesma unga teng kesmaga, burchak unga teng burchakka, uchburchak unga teng uchburchakka, tekislik unga teng tekislikka va tetraedr unga teng tetraedrga ko'chadi (akslanadi).

Fazoda biror harakat yordamida birini ikkinchisiga ko'chirish mumkin bo'lgan shakllar *teng shakllar* deyiladi.

Harakatga eng sodda misol bu parallel ko'chirishdir.

44



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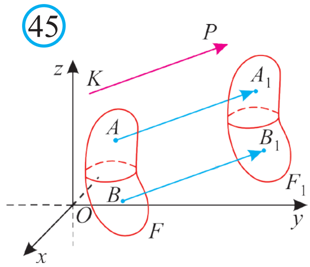

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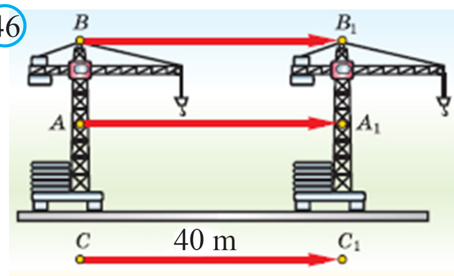


Fazoda biror $\overline{KP}$ vektor va ixtiyoriy X nuqta berilgan bo‘lsin (44- rasm). Agar X_1 nuqta $\overline{XX_1} = \overline{KP}$ shartni qanoatlantirsa, X nuqta X_1 nuqtaga $\overline{KP}$ vektor bo‘ylab *parallel ko‘chirilgan* deb ataladi.

Agar fazoda berilgan F shaklning har bir nuqtasi $\overline{KP}$ vektor bo‘ylab ko‘chirilsa (45- rasm), yangi F_1 shakl hosil bo‘ladi. Bu holda *F shakl F_1 shaklga parallel ko‘chirilgan* deyiladi. Parallel ko‘chirishda F shaklning har bir nuqtasi bir xil yo‘nalishda bir xil masofaga ko‘chirilgan bo‘ladi.

46- rasmda tasvirlangan ko‘tarma kranning har bir nuqtasi boshlang‘ich holatiga nisbatan 40 m ga parallel ko‘chgan.

Ravshanki, parallel ko‘chirish harakatdir. Shuning uchun, parallel ko‘chirishda to‘g‘ri chiziq to‘g‘ri chiziqqa, nur nurga, tekislik tekislikka, kesma unga teng kesmaga ko‘chadi va hokazo.

Aytaylik $\overline{KP} = (a; b; c)$ vektor bo‘ylab parallel ko‘chirishda F shaklning $X\,(x; y; z)$ nuqtasi F_1 shaklning $X_1(x_1; y_1; z_1)$ nuqtasiga o‘tsin. Unda, ta’rifga ko‘ra, quyidagilarga egamiz:

$x_1 - x = a,\ \ y_1 - y = b,\ \ z_1 - z = c$ yoki $x_1 = x + a,\ \ y_1 = y + b,\ \ z_1 = z + c.$

Bu tengliklar *parallel ko‘chirish formulalari* deb ataladi.

1- masala. $\overline{p} = (3; 2; 5)$ vektor bo‘ylab parallel ko‘chirishda $P\,(–2; 4; 6)$ nuqta qaysi nuqtaga ko‘chadi?

Yechish. Yuqoridagi parallel ko‘chirish formulalardan foydalanamiz:

$x_1 = -2 + 3 = 1,\ \ y_1 = 4 + 2 = 6,\ \ z_1 = 6 + 5 = 11.$ *Javob:* $P_1(1; 6; 11).$ □

3.3. Fazoda markaziy simmetriya

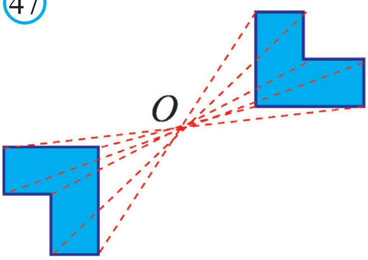
Fazoda berilgan A va A_1 nuqtalar O nuqtaga nisbatan simmetrik deyiladi, agar $\overline{AO} = \overline{OA_1}$ bo‘lsa, ya’ni O nuqta AA_1 kesmaning o‘rtasi bo‘lsa.

Agar fazoda berilgan F shaklning har bir nuqtasi O nuqtaga nisbatan simmetrik nuqtaga ko‘chsa (47- rasm), bunday almashtirishga *O nuqtaga nisbatan simmetriya* deb ataladi. 48, 49- rasmlarda O nuqtaga nisbatan simmetrik shakllar tasvirlangan.

Nuqtaga nisbatan simmetriya – harakatdir.

Agar F shakl O nuqtaga nisbatan simmetrik almashtirishda o‘ziga ko‘chsa, bunday shaklga *markaziy simmetrik shakl* deb ataladi.

47



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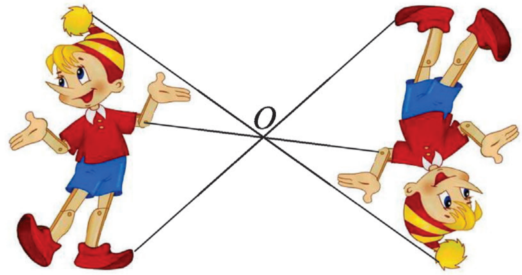
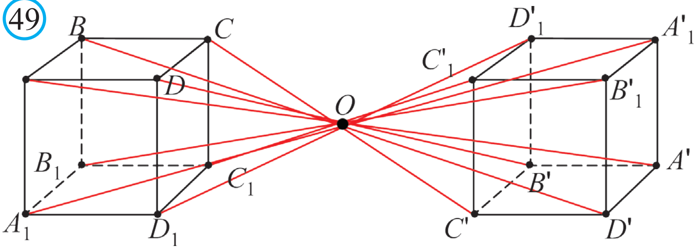


Masalan, parallelepiped (50- rasm) diagonallari kesishish nuqtasi O ga nisbatan markaziy simmetrik shakldir.

49



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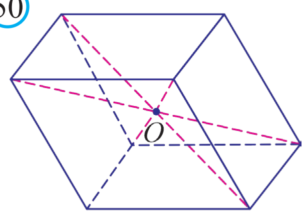


2- masala. O (2; 4; 6) nuqtaga nisbatan markaziy simmetriyada A = (1; 2; 3) nuqta qaysi nuqtaga o‘tadi?

Yechish. $A_1 = (x; y; z)$ izlanayotgan nuqta bo‘lsin. Ta’rifga ko‘ra, O nuqta AA_1 kesmaning o‘rtasi. Demak, $2 = \frac{x+1}{2}$, $4 = \frac{y+2}{2}$, $6 = \frac{z+3}{2}$.

Bu tengliklardan $x = 4 - 1 = 3$, $y = 8 - 2 = 6$, $z = 12 - 3 = 9$.

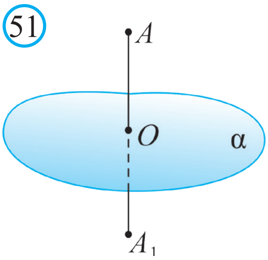
Javob: $A_1(3; 6; 9)$. □

3.4. Tekislikka nisbatan simmetriya

Fazoda berilgan A va A_1 nuqtalar *tekislikka nisbatan simmetrik* deyiladi, agar tekislik AA_1 kesmaga perpendikular bo‘lib, uni teng ikkiga bo‘lsa (51- rasm). 52- rasmda tekislikka nisbatan simmetrik bo‘lgan F_1 va F_2 shakllar keltirilgan. Ravshanki, gavdamiz va aksimiz oyna tekisligiga nisbatan simmetrik bo‘ladi (53- rasm).

Tekislikka nisbatan simmetriya – harakatdir.

51



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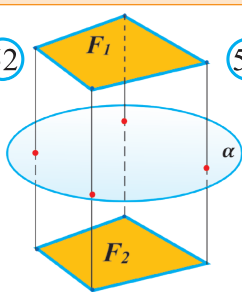

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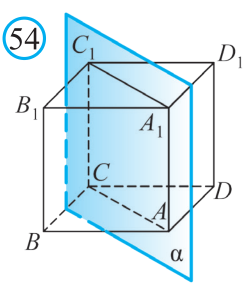


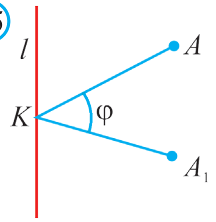
Demak, tekislikka nisbatan simmetriyada kesma o‘ziga teng kesmaga, to‘g‘ri chiziq – to‘g‘ri chiziqqa va tekislik – tekislikka akslanadi.

Agar F shakl tekislikka nisbatan simmetrik almashtirishda o‘ziga ko‘chsa, bunday shaklga *tekislikka nisbatan simmetrik shakl* deyiladi.

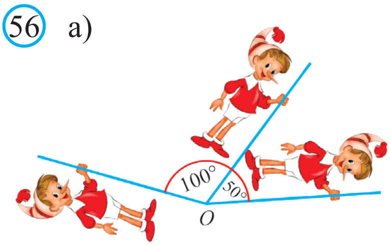
Masalan, 54- rasmda tasvirlangan kub AA_1va CC_1 qirralaridan o‘tuvchi α tekislikka nisbatan simmetrik shakl bo‘ladi.

3.5. Burish va o‘qqa nisbatan simmetriya

55



56 a)



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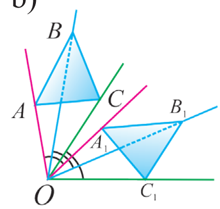

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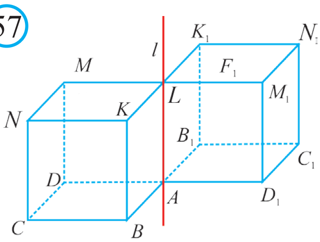


Aytaylik, fazoda A va A_1 nuqtalar va l to‘g‘ri chiziq berilgan bo‘lsin. Agar l to‘g‘ri chiziqqa tushirilgan AK va A_1K perpendikularlar teng va o‘zaro φ burchak tashkil qilsa, bu holda *l to‘g‘ri chiziqqa nisbatan φ burchakka burish* natijasida A nuqta A_1 nuqtaga o‘tdi deyiladi (55- rasm).

Agar fazoda berilgan F shaklning har bir nuqtasi l to‘g‘ri chiziqqa nisbatan φ burchakka bursak, yangi F_1 shakl hosil bo‘ladi. Bunda F shakl l to‘g‘ri chiziqqa nisbatan φ burchakka burishda F_1 shaklga o‘tdi deyiladi. 56- rasmda shunday burishdan hosil bo‘lgan shakllar ko‘rsatilgan.

Masalan, 57- rasmda tasvirlangan kubni l to‘g‘ri chiziqqa nisbatan 180° burchakka burishda yangi kubni hosil qilamiz.

To‘g‘ri chiziqqa nisbatan burish ham harakat bo‘ladi.

l to‘g‘ri chiziqqa nisbatan 180° burchakka burish *l to‘g‘ri chiziqqa nisbatan simmetriya* deb ataladi.

Shaklning simmetriya markazi, o‘qi, tekisligi uning *simmetriya elementlari* deb ataladi.

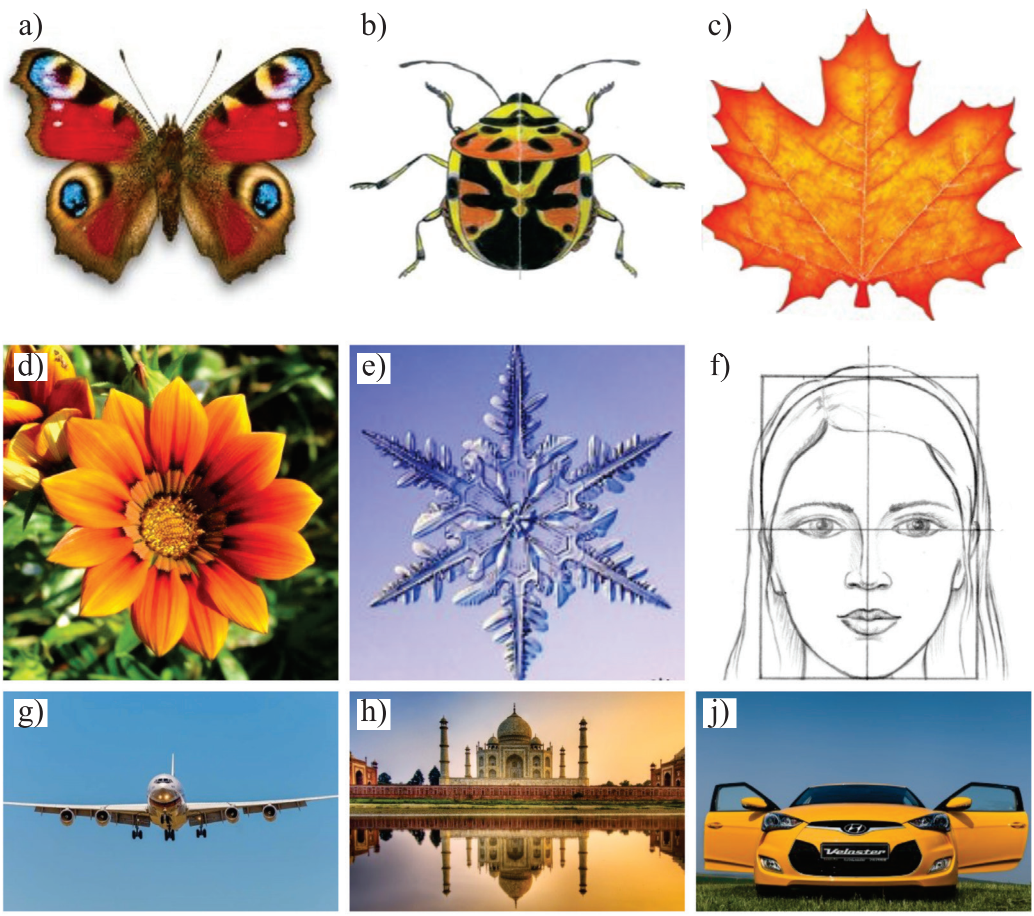
$A\,(x; y; z)$ nuqtaga koordinata tekisliklari, koordinata o‘qlari va koordinata boshiga nisbatan simmetrik nuqtalar quyidagi koordinatalarga ega bo‘ladi:

Simmetriya elementi	**Simmetrik nuqta koordinatalari**
Oxy tekislik	$(x; y; -z)$
Oxz tekislik	$(x; -y; z)$

Oyz tekislik	$(-x; y; z)$
Ox o‘qi	$(x; -y; -z)$
Oy o‘qi	$(-x; y; -z)$
Oz o‘qi	$(-x; -y; z)$
O nuqta	$(-x; -y; -z)$

3.6. Tabiatda va texnikada simmetriya

58

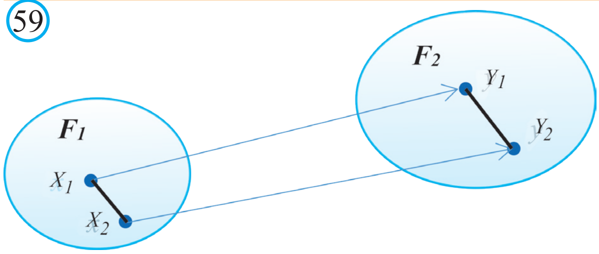


Tabiatda simmetriyani har qadamda uchratish mumkin. Masalan, jonli mavjudodlarning ko‘pchiligi, xususan, inson va hayvonlar gavdasi, o‘simliklarning barglari va gullari simmetrik tuzilgan (58- rasm). Shuningdek, jonsiz tabiat unsurlari ham borki, maslan, qor zarralari, tuz kristallari, moddalarning molekular tuzilishi ham ajoyib simmetrik shakllardan iboratdir. Bu bejiz emas, albatta, chunki simmetrik shakllar chiroyli bo‘lishi bilan birga, qaysidir ma'noda eng maqbul va mukammal hisoblanadi. Shunday ekan, tabiatdagi go'zallik va mukammallik simmetriya asosiga qurilgan, deb aytishimiz mumkin. Tabiatdagi bu go'zallik va mukammallikdan andoza olgan quruvchi, mahandis va arxitektor kabi ijodkorlar yaratgan ko'plab inshoot va binolar, qurilma va mexanizmlar, texnika va transport vositalari ham simmetrik yaratilgan. Bu ishda ularga geometriya fanining yordami beqiyosdir.

3.7. Fazoviy shakllarning o'xshashligi

Fazoda $k \neq 0$ va F_1 shaklni F_2 shaklga akslantiruvchi almashtirish berilgan bo'lsin. Bu akslantirishda F_1 shaklning ixtiyoriy X_1 va X_2 nuqtalari va ular akslangan F_2 shaklning Y_1 va Y_2 nuqtalari uchun $X_1Y_1 = k \cdot X_2Y_2$ bo'lsa, bu almashtirish *o'xshashlik almashtirishi* deb ataladi (59- rasm).

59



60

Ko'rib turganingizdek, fazoda o'xshashlik almashtirishi tushunchasi tekislikdagidek kiritiladi. Shuningdek, uning quyida ko'riladigan qator turlari ta'rifi, ularning xossalari va bu xossalarning isboti ham tekislikdagisiga o'xshash. Shu bois, bu xossalarning isbotiga to'xtalmaymiz va ularni mustaqil bajarishni tavsiya qilamiz.

Fazodagi o'xshashlik almashtirishi to'g'ri chiziqni to'g'ri chiziqqa, nurni nurga, kesmani kesmaga va burchakni burchakka akslantiradi. Shuningdek, bu almashtirish tekislikni ham tekislikka akslantiradi.

Fazoda berilgan ikki shaklning biri ikkinchisiga o'xshashlik almashtirishi orqali akslansa, ular *o'xshash shakllar* deb ataladi.

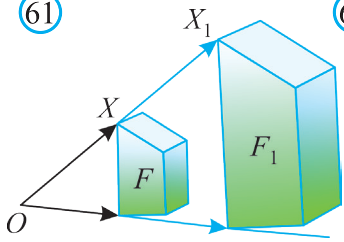
Fazoda F shakl, O nuqta va k noldan farqli ($k \neq 0$) son berilgan bo'lsin.

F shaklning ixtiyoriy X nuqtasini $\overline{OX_1} = k\overline{OX}$ shartni qanoatlantiruvchi X_1 nuqtaga akslantiruvchi almashtirish *O nuqtaga nisbatan k koeffitsiyentli gomotetiya* deb ataladi (61-rasm). O nuqtaga *gomotetiya markazi*, k soniga esa *gomotetiya koeffitsiyenti* deyiladi.

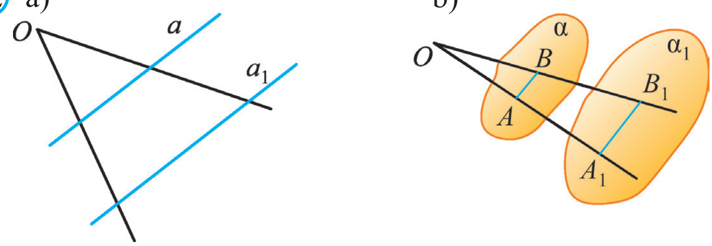
F shaklning har bir nuqtasi shu usulda akslantirilsa, natijada F_1 shakl hosil bo'ladi va bu *gomotetiyada F shakl F_1 shaklga akslanadi* deyiladi.

Ko'rib turganingizdek, fazoda gomotetiya ta'rifi tekislikdagisi bilan deyarli bir xil. Shuningdek, uning qator xossalari ham borki, ular ham, ularning isbotlari ham tekislikdagisiga o'xshash. Shu bois, bu xossalarning isbotiga to'xtalmaymiz va ularni mustaqil bajarishni tavsiya qilamiz.

61



62 a) b)



O nuqtaga nisbatan k koeffitsiyentli gomotetiya o'xshashlik almashtirishidir.

Gomotetiya koeffitsiyenti *k* ixtiyoriy noldan farqli son bo'lib, $k=1$ da *F* shakl o'ziga o'zi akslanadi, $k=-1$ da esa *F* shakl *O* nuqtaga nisbatan simmetrik F_1 shaklga akslanadi. Boshqa hollarda gomotetiya nuqtalar orasidagi masofani saqlamaydi, ya'ni u harakat bo'lmaydi. Gomotetiya natijasida nuqtalar orasidagi masofa bir xil *k* songa ko'payadi, ya'ni shaklning o'lchamlari o'zgaradi, lekin uning shakli o'zgarmaydi.

Gomotetiyada gomotetiya markazidan o'tmaydigan a) to'g'ri chiziq unga parallel to'g'ri chiziqqa (62.a- rasm); b) tekislik esa unga parallel tekislikka akslanadi (62.b- rasm).

Gomotetiyada gomotetiya markazidan o'tuvchi to'g'ri chiziq yoki tekislik o'ziga o'zi akslanadi.

Mavzuga oid masalalar va amaliy topshiriqlar

83. $\overline{p}$ = (–2; 1; 4) vektor bo'ylab parallel ko'chirishda, a) (3; –2; 3); b) (0; 2; –3); c) (2; –5; 0) nuqta qaysi nuqtaga ko'chadi?

84. Parallel ko'chirishda *A*(4; 2; –8) nuqta (3; 7; –5) nuqtaga ko'chdi. Parallel ko'chirish qaysi vektor bo'ylab amalga oshirilgan?

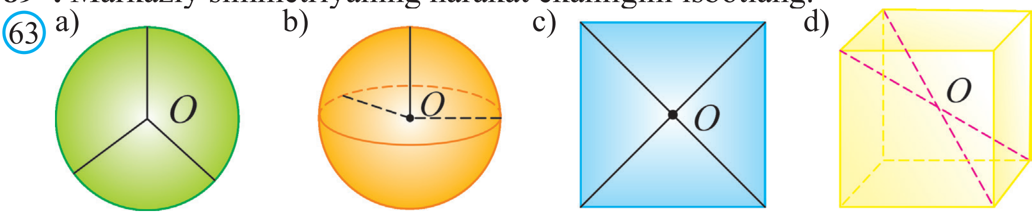
85. Parallel ko'chirishda: a) to'g'ri chiziq - to'g'ri chiziqqa; b) nur nurga; c) tekislik tekislikka; d) kesma unga teng kesmaga ko'chishini isbotlang.

86. *O* (–2; 3; –1) nuqtaga nisbatan markaziy simmetriyada *A* (4; 2; –3) nuqta qaysi nuqtaga o'tadi?

87. 63- rasmda tasvirlangan shakllarda *O* nuqta simmetriya markazi ekanligini asoslang.

88. (–2; 5; –9), (2; 2; –7), (–6; 12; –2) nuqtalar koordinata boshiga nisbatan markaziy simmetriyada qaysi nuqtalarga o'tadi?

89*. Markaziy simmetriyaning harakat ekanligini isbotlang.



90*. Tekislikka nisbatan simmetriyaning harakat ekanligini isbotlang.

91. Parallelepipedning (50- rasm) diagonallari kesishish nuqtasi O ga nisbatan markaziy simmetrik shakl ekanligini isbotlang.

92. (1; 2; –3), (0; 2; –3), (2; 2; –3) nuqtalar koordinata tekisliklariga nisbatan simmetriyalarda qaysi nuqtalarga o'tadi?

93. (2; 4; –1) nuqta koordinata tekisligiga nisbatan simmetrik akslantirishda (2; –4; –1) nuqtaga o'tdi. Akslantirish qaysi koordinata tekisligiga nisbatan amalga oshirilgan?

94. Quyidagi jadvalda berilgan 1- namuna asosida bo'sh kataklarni to'ldiring.

№	Berilgan nuqta	Simmetrik nuqta	Nimaga nisbatan simmetrik?
1	(1; 2; 3)	(1; 2; –3)	*Oxy* tekislikka nisbatan
2	(2; 4; –1)		*Oxz* tekislikka nisbatan
3		(1; 2; 3)	*Oyz* tekislik
4	(–1; –2; –3)	(–1; 2; 3)	
5	(–1; 6; 3)		*Oy* o'qi
6		(–3; 8; –2)	*Oz* o'qi
7	(4; 1; –2)		*O* nuqta

95. 49- rasmda tasvirlangan shakllarda O nuqta simmetriya markazi ekanligini asoslang.

96*. To'g'ri chiziqqa nisbatan burish harakat ekanligini ko'rsating.

97. O nuqtaga nisbatan k koeffitsiyentli gomotetiya o'xshashlik almashtirishi ekanligini ko'rsating.

98. Oxy tekislikka nisbatan simmetriyada ixtiyoriy $(x; y; z)$ nuqta $(x; y; -z)$ nuqtaga o'tishini ko'rsating.

99. Oxz tekislikka nisbatan simmetriyada ixtiyoriy $(x; y; z)$ nuqta $(x; -y; z)$ nuqtaga o'tishini ko'rsating,

100. Parallel ko'chirishda (1; 2; –1) nuqta (1; –1; 0) nuqtaga o'tdi. Koordinata boshi bu almashtirishda qaysi nuqtaga o'tadi?

101. Parallel ko'chirishda (3; 4; –1) nuqta (2; –4; 1) nuqtaga o'tdi. Bu almashtirishda koordinata boshi qaysi nuqtaga o'tadi?

102*. $A(2; 1; 0)$ nuqta B (1; 0; 1) nuqtaga, C (3; –2; 1) nuqta esa D (2; –3; 0) nuqtaga oʻtadigan parallel koʻchirish mavjudmi?

103*. A (–2; 3; 5) nuqta B (1; 2; 4) nuqtaga, C (4; –3; 6) nuqta esa D(7; –2; 5) nuqtaga oʻtadigan parallel koʻchirish mavjudmi?

104. 58- rasmda tasvirlangan jonli va jonsiz obyektlar fazoviy jism sifatida qanday simmetrik shakl boʻlishi mumkinligini aniqlang.Ularning (agar mavjud boʻlsa) simmetriya markazi, simmetriya oʻqi yoki simmetriya tekisliklarini chizib koʻrsating.

105. 60- rasmda tasvirlangan ona-bolalar (matreshkalar) ning katta ona matreshkaga nisbatan oʻxshashlik koeffitsiyentlarini aniqlang.

106. Muntazam tetraedr qirrasining uzunligi 12 cm ga teng. Bu tetraedrga: a) 3; b) –4; c) $\frac{1}{2}$; d) $-\frac{1}{3}$; koeffitsiyentli gomotetik boʻlgan tetraedr qirrasining uzunligi nimaga teng?

107. Ixtiyoriy ABC uchburchak chizing va biror O nuqtani belgilang. Markazi O nuqtada va koeffitsiyenti: a) 2; b) –3; c) $-\frac{1}{2}$; d) $\frac{1}{4}$ ga teng boʻlgan gomotetiyada ABC uchburchak oʻtadigan uchburchakni quring.

108. Ixtiyoriy $SABC$ tetraedr chizing. Markazi S nuqtada va koeffitsiyenti: a) 1,5; b) –2; c) $\frac{1}{2}$; d) $\frac{1}{4}$ ga teng boʻlgan gomotetiyada $SABC$ tetraedr oʻtadigan tetraedrni quring.

109. Ixtiyoriy kub chizing. Markazi kubning biror uchida va koeffitsiyenti: a) 2; b) –2; c) $\frac{1}{2}$; d) $-\frac{1}{2}$ ga teng boʻlgan gomotetiyada bu kub oʻtadigan fazoviy geometrik shaklni quring.

110. Markazi koordinata boshida va koeffitsiyenti: a) 2,5; b) –2,5; c) $\frac{1}{4}$; d) $\frac{1}{4}$ ga teng boʻlgan gomotetiyada $A(–2; 3; 5)$ nuqta oʻtadigan nuqtaning koordinatalarini toping.

111. Markazi $O(-1; 2; 2)$ nuqtada va koeffitsiyenti: a) 0,5; b) –2; c) $\frac{1}{4}$; d) $-\frac{1}{4}$ ga teng boʻlgan gomotetiyada $A(2; 4; 0)$ nuqta oʻtadigan nuqtaning koordinatalarini toping.

112. Uchlari $O(0; 0; 0)$, $A(4; 0; 0)$, $B(0; 4; 0)$, $C(0; 0; 4)$ nuqtalarda boʻlgan tetraedr: a) markazi O nuqtada, koeffitsiyenti –1 ga teng; b) markazi A nuqtada, koeffisiyenti 2 ga teng boʻlgan gomotetiyada oʻtadigan tetraedrning uchlari koordinatalarini toping.

113*. Gomotetiyada uning markazidan oʻtmaydigan: a) toʻgʻri chiziq oʻziga parallel toʻgʻri chiziqqa, b) tekislik esa oʻziga parallel tekislikka akslanishini koʻrsating.

114*. Gomotetiyada uning markazidan oʻtuvchi toʻgʻri chiziq yoki tekislik oʻziga oʻzi akslanishini koʻrsating.

4. BOBNI TAKRORLASHGA DOIR AMALIY MASHQLAR

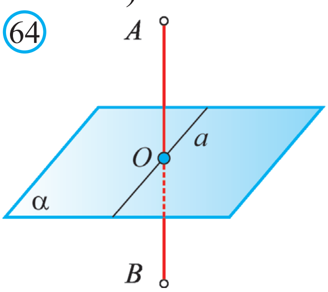
4.1. 1- test sinovi

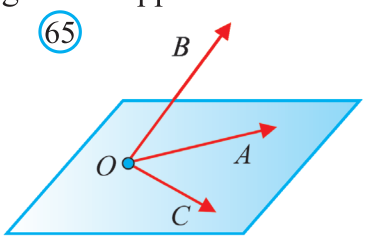
1. $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalar berilgan. $z_2 - z_1$ nimani anglatadi?

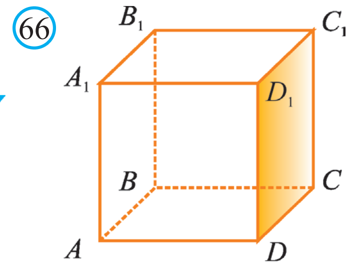
A) AB kesma o‘rtasining koordinatasini; B) AB kesma uzunligini;
C) $\overline{AB}$ vektor uzunligini; D) $\overline{AB}$ vektor koordinatalaridan birini.

2. 64- rasmda $AB \perp \alpha$, $a \subset \alpha$, $AO = OB$ bo‘lsa,

A) A va B nuqtalar O nuqtaga nisbatan simmetrik bo‘ladi;
B) A va B nuqtalar a to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi;
C) A va B nuqtalar α tekislikka nisbatan simmetrik bo‘ladi;
D) AB kesma a to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi.







3. 65- rasmda B nuqta AOC tekislikda yotmaydi. Unda $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ vektorlar …

A) kollinear; B) komplanar;
C) bir xil yo‘nalishli; D) komplanar emas.

4. $M(-7; 1; 4)$ va $N(-1; -3; 0)$ nuqtalar berilgan. MN kesma o‘rtasining koordinatalarini toping.

A) $(-4; -1; 4)$; B) $(-4; -1;2)$; C) $(-4; -2; 2)$; D) $(-3; 2; 2)$.

5. $A(0; -3; 2)$ va $B(4; 0; -2)$ nuqtalar berilgan. AB kesma o‘rtasi nimaga tegishli?

A) Ox o‘qiga; B) Oy o‘qiga; C) Oz o‘qiga; D) Oxy tekisligiga.

6. $A(3; 4; -3)$ nuqtadan Oz o‘qigacha bo‘lgan masofani toping.

A) 3; B) 5; C) $2\sqrt{3}$; D) $\sqrt{34}$.

7. $\overline{CD}+\overline{DE}+\overline{EF}$ vektorlar yig‘indisini toping.

A) $\overline{O}$; B) $\overline{CF}$; C) $\overline{DF}$; D) $\overline{CE}$.

8. m ning qaysi qiymatida $\overline{a}(m; 4; -3)$ va $\overline{b}(4; 8; -6)$ vektorlar kollinear bo‘ladi?

A) 2; B) 5; C) 1; D) 3.

9. O nuqta α tekislikda yotmaydi. Markazi O nuqtada bo‘lgan gomotetiyada α tekislik undan farqli bo‘lgan β tekislikka o‘tadi. Agar a to‘g‘ri chiziq α tekislikka tegishli bo‘lsa, …

A) $\alpha \parallel \beta$ bo‘ladi; B) α tekislik β tekislik bilan kesishadi;
C) $a \subset \beta$ bo‘ladi; D) $\alpha \perp \beta$ bo‘ladi.

10. AB to‘g‘ri chiziq BCD tekislikka perpendikular. Qaysi vektorlarning skalar ko‘paytmasi nolga teng bo‘ladi?

A) $\overline{CA}$ va $\overline{CB}$; B) $\overline{BD}$ va $\overline{AD}$; C) $\overline{AC}$ va $\overline{BC}$; D) $\overline{AB}$ va $\overline{CD}$.

11. Qirrasi 1 ga teng bo‘lgan $ABCDA_1B_1C_1D_1$ kub berilgan (66- rasm). $(\overline{AB}+\overline{BC})\cdot\overline{BB}$ ni toping.

A) 1; B) 0; C) –1; D) 0,5.

12. p ning qaysi qiymatida $\overline{a}(1;\ 1;\ 0)$ va $\overline{b}(0;\ 4;\ p)$ vektorlar orasidagi burchak 60° ga teng bo‘ladi?

A) 4; B) 4 yoki –4; C) 16; D) 16 yoki –16.

13. $ABCDA_1B_1C_1D_1$ kub berilgan. Parallel ko‘chirishda A_1D kesma D_1C kesmaga o‘tadi. Bu ko‘chirishda AA_1B_1 tekislik qaysi tekislikka o‘tadi?

A) DB_1B; B) DCC_1; C) AA_1C_1; D) ABC.

14. α tekislik unda yotmaydigan ABC uchburchakning simmetriya tekisligidir. Qaysi tasdiq to‘g‘ri?

A) $(ABC)\perp\alpha$; B) ABC uchburchak teng yonli;
C) ABC uchburchakning simmetriya markazi bor;
D) ABC uchburchakning simmetriya o‘qi bor.

15. $ABCDA_1B_1C_1D_1$ kub berilgan. $\overline{A_1B_1}+\overline{BC}-\overline{DD_1}$ ni toping.

A) $\overline{A_1C}$; B) $\overline{BD_1}$; c) $\overline{B_1D}$; D) $\overline{AC_1}$.

16. Qaysi geometrik almashtirish ikki ayqash to‘g‘ri chiziqlardan birini ikkinchisiga o‘tkazadi?

A) parallel ko‘chirish; B) tekislikka nisbatan simmetriya;
C) burish; D) gomotetiya.

17. $M(-1;\ 2;\ -4)$ nuqtaga Oyz tekislikka nisbatan simmetrik bo‘lgan nuqtani toping.

A) (1; –2; 4); B) (1; 2; –4); C) (–1; –2; –4); D) (–1; 2; 4).

18. Parallel ko‘chirishda $\overline{AB}$ vektor $\overline{DC}$ vektorga o‘tadi. Qaysi tasdiq noto‘g‘ri?

A) $\overline{AB}=\overline{DC}$; B) AC va BD kesma o‘rtalari ustma-ust tushadi;
C) $\overline{AB}$, $\overline{AC}$ va $\overline{DC}$ vektorlar komplanar; D) $ABCD$ parallelogramm.

19. $B(-3;\ 2;\ -5)$ nuqta Oxz tekislikdan qanday masofada yotibdi?

A) 2; B) 5; C) 3; D) $\sqrt{34}$.

20. $A(1;\ -2;\ 0)$, $B(1;\ -4;\ 2)$, $C(3;\ 2;\ 0)$ nuqtalar ABC uchburchakning uchlari. CM mediana uzunligini toping.

A) $2\sqrt{3}$; B) $3\sqrt{2}$; C) $\sqrt{6}$; D) 18.

21. Agar $\overline{a}(1;\ m;\ 2)$ va $\overline{b}(0{,}5m+1;\ 3;\ 1)$ vektorlar kollinear bo‘lsa, $m+n$ ni toping.

A) 3; B) 5; C) –4; D) 9.

22. $A(-1;\ -9;\ -3)$ va $B(0;\ -2;\ 1)$ nuqtalar berilgan. vektorni koordinata vektorlari (ortlar) bo‘yicha yoying.

A) $(\overline{BA}) = \bar{i} + 9\bar{j} - \bar{k}$; B) $(\overline{BA}) = \bar{i} - 9\bar{j} + \bar{k}$;
C) $(\overline{BA}) = -\bar{i} - 9\bar{j} - 4\bar{k}$; D) $(\overline{BA}) = \bar{i} + 9\bar{j} - 4\bar{k}$.

23. $A(1; –2; 2)$, $B(1; 4; 0)$, $C(–4; 1; 1)$ va $D(–5; –5; 3)$ nuqtalar berilgan. AC va BD vektorlar orasidagi burchakni toping
A) 150°; B) 30°; C) 45°; D) 90°.

24. $|\bar{a}| = 6$, $|\bar{a}+\bar{b}| = 11$, $|\bar{a}-\bar{b}| = 7$ ekanligi ma'lum bo'lsa, $|\bar{b}|$ ni toping.
A) 11; B) 18; C) 20 ; D) 7.

25. Asoslari BC va AD bo'lgan $ABCD$ trapetsiya berilgan. Agar $\overline{AB}(–7; 4; 5)$, $\overline{AC}(3; 2; –1)$, $\overline{AD}(20; –4; –12)$, M va N – mos ravishda AB va CD tomonlar o'rtasi bo'lsa, $\overline{MN}$ vektor koordinatalari yig'indisini toping.
A) 1; B) 2 ; C) 3; D) 4.

4.2. Masalalar

115. Uchlari $A(1; –2; 4)$ va $B(3; –4; 2)$ nuqtalarda bo'lgan kesma o'rtasining koordinatalarini toping.

116. $A(x; 0; 0)$ nuqta $B(1; 2; 3)$ va $C(–1; 3; 4)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, x ni toping.

117. Agar kesmaning bir uchi $A(1; –5; 4)$, o'rtasi $C(4; –2; 3)$ nuqtada bo'lsa, ikkinchi uchining koordinatalari qanday bo'ladi?

118. Oxz tekisligiga nisbatan $A(1; 2; 3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

119. Koordinatalar boshiga nisbatan $A(1; 2; 3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

120. Oxy tekisligiga nisbatan (1; 2; 3) nuqtaga simmetrik bo'lgan nuqtani toping.

121. Oy o'qqa nisbatan (2; –3; 5) nuqtaga simmetrik bo'lgan nuqtani toping.

122. Quyidagi nuqtalardan qaysi biri Oyz tekislikda yotadi?
$A(2; –3; 0)$; $B(2; 0; –5)$; $C(1; 0; – 4)$; $D(0; 9; –7)$; $E(1; 0; 0)$.

123. Quyidagi nuqtalardan qaysi biri Oxz tekislikda yotadi:
$A(– 4; 3; 0)$; $B(0; –7; 0)$; $C(2; 0; –8)$; $D(2; –4; 6)$; $E(0; – 4; 5)$?

124. $A(–3; 8; 3\sqrt{33})$ nuqtadan Ox o'qqacha bo'lgan masofani toping.

125. $A(3; –2; 5)$ va $B(–4; 5; –2)$ nuqtalar berilgan. $\overline{AB}$ vektorning koordinatalarini toping.

126. $\bar{a}(1; –2; 3)$ vektorning oxiri $B(2; 0; 4)$ nuqta bo'lsa, bu vektorning boshini toping.

127. $B(0; 4; 2)$ nuqta $\bar{a}(2; –3; 1)$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

128. $\bar{a}(x; 1; 2)$ vektorning uzunligi 3 ga teng. x ning qiymatini toping.

129. $\bar{a}(4; –12; z)$ vektorning moduli 13 ga teng. z ning qiymatini toping.

130. Agar $\bar{a}(6; 2; 1)$ va $\bar{b}(0; –1; 2)$ bo'lsa, $\bar{c} = 2\bar{a} - \bar{b}$ vektorning uzunligini toping.

131. Agar $\overline{p}(2; 5; -1)$ va $\overline{q}(-2; 2)$ bo‘lsa, $\overline{m}= 4\overline{p}+2\overline{q}$ vektorning uzunligini toping.

132. $\overline{a}(2; -3; 4)$ va $\overline{b}(-2; -3; 1)$ vektorlarning skalar ko‘paytmasini toping.

133. $\overline{m}(-1; 5; 3)$ va $\overline{n}(2; -2; 4)$ vektorlarning skalar ko‘paytmasini toping.

134. m ning qanday qiymatida $\overline{a}(1; m; -2)$ va $\overline{b}(m; 3; -4)$ vektorlar perpendikular bo‘ladi?

135. n ning qanday qiymatida $\overline{a}(n; -2; 1)$ va $\overline{b}(n; n; 1)$ vektorlar perpendikular bo‘ladi?

136. m ning qanday qiymatida $\overline{a} = m\overline{i} + 3\overline{j} + 4\overline{k}$ va $\overline{b} = 4\overline{i} + m\overline{j} - 7\overline{k}$ vektorlar perpendikular bo‘ladi?

137. $A(1; -2; 2)$, $B(1; 4; 0)$, $C(-4; 1; 1)$ va $D(-5; -5; 3)$ nuqtalar berilgan. $\overline{AC}$ va $\overline{BD}$ vektorlar orasidagi burchakni toping.

138. n ning qanday qiymatlarida $\overline{a}(2; n; 6)$ va $\overline{b}(1; 2; 3)$ vektorlar kollinear bo‘ladi?

139. m ning qanday qiymatida $\overline{a}(2; 3; -4)$ va $\overline{b}(m; -6; 8)$ vektorlar parallel bo‘ladi?

140. m va n ning qanday qiymatida $\overline{a}(-1; m; 2)$ va $\overline{b}(-2; -4; n)$ vektorlar kollinear bo‘ladi?

141. $A(2; 7; -3)$ va $B(-6; -2; 1)$ nuqtalar berilgan. $\overline{BA}$ vektorni koordinatalar vektorlari (ortlari) bo‘yicha yoying.

4.3. 1- nazorat ishi namunasi

1. Oxy tekisligiga nisbatan (1; 2; 3) nuqtaga simmetrik bo‘lgan nuqtani toping.
2. Agar $\overline{a}(6; 3; 2)$ va $\overline{b}(-3; 1; 5)$ bo‘lsa, $\overline{c} = \overline{a} + 2\overline{b}$ vektorning uzunligini toping.
3. $A(2; -1; 0)$ va $B(-2; 3; 2)$ nuqtalar berilgan. Koordinata boshidan AB kesma o‘rtasigacha bo‘lgan masofani toping
4. $A(1; -2; 2)$, $B(1; 4; 0)$, $C(-4; 1; 1)$ va $D(-5; -5; 3)$ nuqtalar berilgan. $\overline{AC}$ va $\overline{BD}$ vektorlar orasidagi burchakni toping.
5. *(Yaxshi o‘zlashtiradigan o‘quvchilar uchun qo‘shimcha masala).* Uchlari $A(4; 5; 1)$, $B(2; 3; 0)$ va $C(2; 1; -1)$ nuqtalarda bo‘lgan uchburchakning BD medianasi uzunligini toping

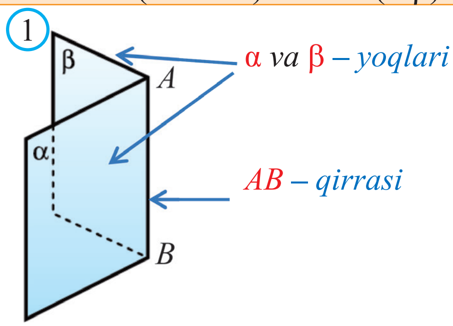
I BOB. PRIZMA VA SILINDR

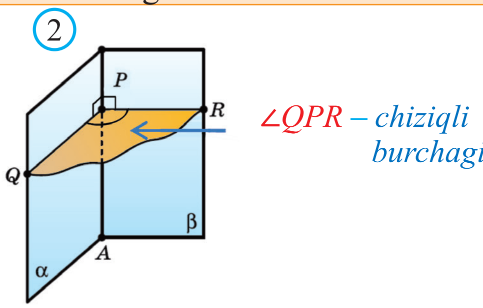
5. KO‘PYOQLI BURCHAKLAR VA KO‘PYOQLAR

5.1. Ko‘pyoqli burchaklar

Ikkiyoqli burchak bilan 10- sinfda tanishgansiz.

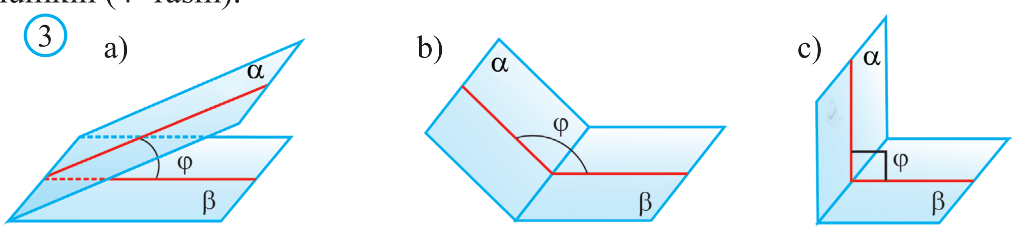
Ikkita α va β yarimtekislik (*yoqlari*) va ularni chegaralab turgan umumiy *AB* to‘g‘ri chiziq (*qirrasi*) dan iborat geometrik shakl *ikkiyoqli burchak* deb ataladi (1- rasm) hamda (α β) tarzda belgilanadi.

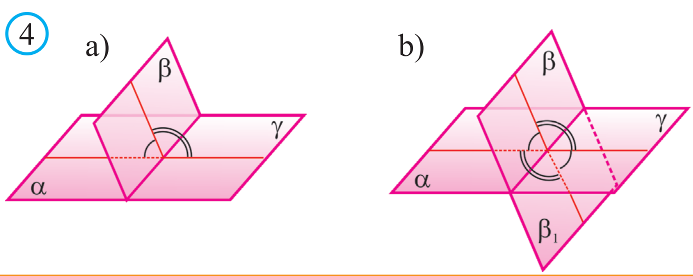


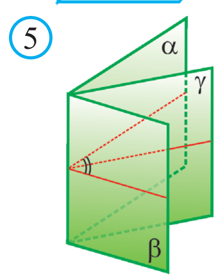


Ikkiyoqli burchak qirrasining ixtiyoriy *P* nuqtasi dan uning yoqlarida yotuvchi va bu qirraga perpendikular bo‘lgan *PR* va *PQ* nurlarni chiqaramiz. ∠*QPR* – ikkiyoqli burchakning *chiziqli burchagi* deb ataladi (2- rasm).

Ikkiyoqli burchaklar yassi burchaklar kabi chiziqli burchagining kattaligiga qarab *o‘tkir, o‘tmas, to‘g‘ri va yoyiq* bo‘ladi (3- rasm). Yassi burchaklar kabi ikkita ikkiyoqli burchaklar *qo‘shni va vertikal* bo‘lishi mumkin (4- rasm).



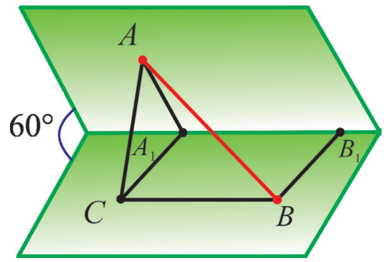




Ikkiyoqli burchakni teng ikkiga bo‘luvchi yarimtekislik uning *bissektori* deb ataladi (5- rasm).

1- masala. Chiziqli burchagi 60° ga teng bo‘lgan ikkiyoqli burchakning yoqlarida yotgan A va B nuqtalardan (6- rasm) uning qirrasiga AA_1 va BB_1 perpendikularlar tushirilgan. Agar $AA_1 = 12$, $BB_1 = 10$ va $A_1B_1 = 13$ bo‘lsa, AB kesma uzunligini toping.

6



Yechish. $BB_1 || CA_1$ va $A_1B_1 || CB$ to‘g‘ri chiziqlarni o‘tkazamiz. Hosil bo‘lgan A_1B_1BC to‘rtburchak parallelogramm bo‘ladi. A_1B_1 to‘g‘ri chiziq A_1AC uchburchak tekisligiga perpendikular bo‘ladi, chunki u bu tekislikda yotgan ikkita A_1A va A_1C to‘g‘ri chiziqlarga perpendikular. Unda BC to‘g‘ri chiziq ham bu tekislikka perpendikular bo‘ladi.

Demak, ABC uchburchak to‘g‘ri burchakli uchburchak ekan.

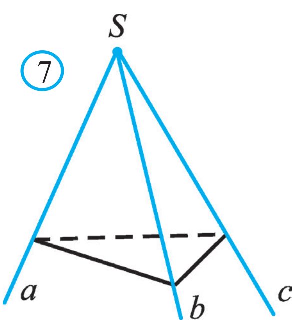
Kosinuslar teoremasiga ko‘ra:

$$AC^2 = AA_1^2 + A_1C^2 - 2AA_1 \cdot A_1C \cdot \cos\alpha = 12^2 + 10^2 - 2 \cdot 12 \cdot 10 \cdot \cos 60° = 124.$$

Pifagor teoremasiga ko‘ra: $AB = \sqrt{AC^2 + BC^2} = \sqrt{124 + 169} = \sqrt{293}.$

Javob: $AB = \sqrt{293}$. □

7



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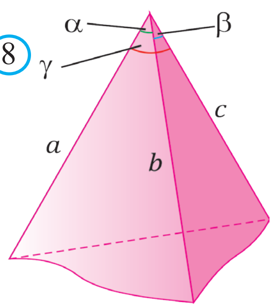

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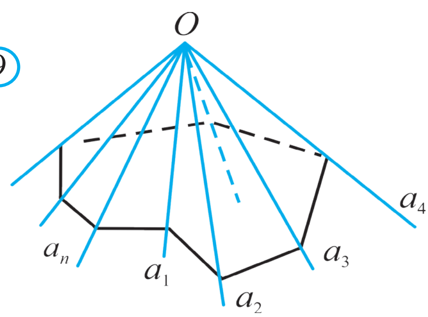


Fazoda bir nuqtadan chiquvchi *a, b* va *c* nurlar uchta yassi *(ab), (bc)* va *(ac)* burchaklar tashkil qiladi (7- rasm). Bu yassi burchaklardan tashkil topgan *(abc)* shaklga *uchyoqli burchak* deyiladi. Yassi burchaklarga uchyoqli burchakning *yoqlari*, ularning tomonlariga uchyoqli burchakning *qirralari,* umumiy uchiga esa uchyoqli burchakning *uchi* deyiladi.

Uchyoqli burchakning yoqlaridan tashkil qilgan ikkiyoqli burchaklar uchyoqli burchakning *ikkiyoqli burchaklari* deb ataladi.

Uchta yassi *(ab), (bc)* va *(ac)* burchaklar uchyoqli burchakning *tekis burchaklari* deb ham yuritiladi.

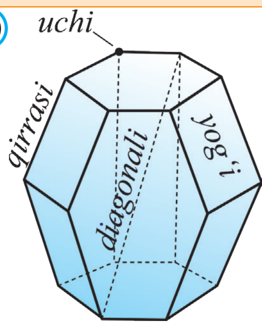
Uchyoqli burchakning tekis burchaklarini, mos ravishda, α, β, γ deb belgilasak (8- rasm), ular uchun uchburchak tengsizligi o‘rinli bo‘ladi, ya'ni ularning ixtiyoriysi qolgan ikkitasining yig'indisidan kichik bo'ladi: $\alpha+\beta<\gamma$, $\alpha+\gamma<\beta$, $\beta+\gamma<\alpha$ va tekis burchaklarining yig'indisi 360° dan kichik bo'ladi: $\alpha+\beta+\gamma<360^{\circ}$.

Ko'pyoqli burchak tushunchasi ham shunga o'xshash kiritiladi (9- rasm).

5.2. Ko'pyoqlar

E'tibor bergan bo'lsangiz, shu choqqacha fazoviy shakl sifatida qator jismlarning, xususan ko'pyoqlarning xossalarini o'rganib keldik. Bu fazoviy shakllarning *jism* deb atalishiga sabab, ularni fazoning biror moddiy jism egallagan va sirt bilan chegaralangan bo'lagi sifatida tasavvur etish mumkinligidir. Quyida ko'pyoqlarga tegishli ba'zi tushunchalarni eslatib o'tamiz.

Ko'pyoq deb yassi ko'pburchaklar bilan chegaralangan jismga aytiladi (10- rasm).

10


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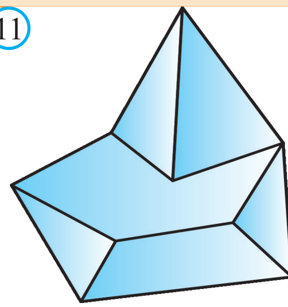
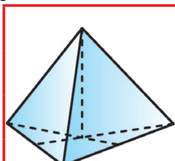
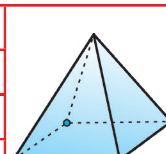
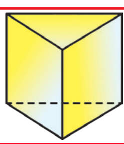
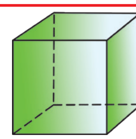


Ko'pyoq ixtiyoriy yog'i yotgan tekislikning bir tomonida yotsa, bunday ko'pyoq *qavariq ko'pyoq* deyiladi. 10- rasmda qavariq, 11- rasmda esa qavariq bo'lmagan ko'pyoqlar tasvirlangan.

Ixtiyoriy qavariq ko'pyoqning yoqlari sonini *Y*, uchlari sonini *U* va qirralari sonini *Q* bilan belgilaylik. Bizga ma'lum ko'pyoqlar uchun quyidagi jadvalni to'ldiraylik:

	**Ko'pyoq nomi**	**Y**	**U**	**Q**	
	Uchburchakli piramida	4	4	6	
	To'rtburchakli piramida	5	5	8	
	Uchburchakli prizma	5	6	9	
	To'rtburchakli prizma	6	8	12	
	n- burchakli piramida	$n+1$	$n+1$	$2n$	
	n- burchakli prizma	$n+2$	$2n$	$3n$	

Jadvaldan har bir ko'pyoq uchun $Y + U - Q = 2$ bo'lishini payqash mumkin. Ma'lum bo'lishicha, bu munosabat barcha qavariq ko'pyoqlar uchun to'g'ri bo'lar ekan. Buni ilk bor 1752- yilga shvetsariyalik matematik Leonard Eyler aniqlagan.

Eyler teoremasi. Ixtiyoriy qavariq ko‘pyoq uchun: $Y + U - Q = 2$ munosabat o‘rinli bo‘ladi, bu yerda Y– ko‘pyoqning yoqlari, U– uchlari, Q– esa qirralari soni.

Bu teoremaning isbotiga to‘xtalmaymiz. Undan quyidagi natijalar kelib chiqadi. Ularni Eyler teoremasidan foydalanib mustaqil isbotlang.

1- natija. Ko‘pyoq tekis burchaklarining soni uning qirralari sonidan ikki marta ko‘p.

2- natija. Ko‘pyoq tekis burchaklari soni har doim juft bo‘ladi.

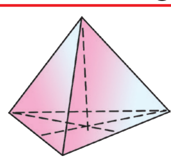
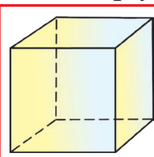
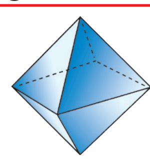
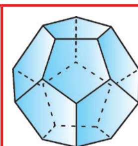
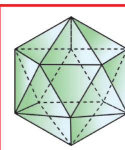
3- natija. Agar ko‘pburchakning har bir uchida bir xil k sondagi qirralar tutashsa, $U \cdot k = 2Q$ munosabat o‘rinli bo‘ladi.

4- natija. Agar ko‘pyoqning barcha yoqlari bir xil n-burchaklardan tashkil topgan bo‘lsa, $Y = 2Q$ munosabat o‘rinli bo‘ladi.

5- natija. Ko‘pyoqning tekis burchaklari yig‘indisi $360°(Y - Q)$ ga teng.

Yoqlari bir-biriga teng muntazam ko‘pburchaklardan iborat va har bir uchidan bir xil sondagi qirralar chiqadigan qavariq ko‘pyoqli *muntazam ko‘pyoqli* deb ataladi.

Ma’lum bo‘lishicha muntazam ko‘pyoqlilar besh xil bo‘lar ekan (buni mustaqil tekshirib ko‘ring). Bular quyidagilar:

Shakli					
Nomi va uning talqini	muntazam tetraedr (to‘rtyoqli)	Kub, geksaedr (oltiyoqli)	Oktaedr (sakkiz-yoqli)	Dode-kaedr (o‘n ikkiyoqli)	Ikosaedr (yigirma-yoqli)
Yoqlari	muntazam uchburchak	munta-zam to‘rt-burchak	muntazam uchburchak	munta-zam besh-burchak	munta-zam uch-burchak
Yoqlari soni	4	6	8	12	20
Qirralari soni	6	12	12	30	30
Uchlari soni	4	8	6	20	12
Har bir uchdan chiquvchi qirralar soni	3	3	4	3	5

Tarixiy ma'lumotlar

Barcha muntazam ko'pyoqlar Qadimgi Yunonistonda ma'lum edi. Yevklidning mashhur "Negizlar"ining XIII kitobi muntazam ko'pyoqlarga bag'ishlangan. Bu ko'pyoqlarni ko'pincha Platon jismlari deb ataladi. Qadimgi Yunonistonning buyuk olimi Platon (miloddan oldingi 427–347- yillar) bayon qilgan olamning idealistik tasvirida bu jismlardan to'rttasi olamning to'rt unsuriga (elementiga) o'xshatilgan: tetraedr – olov, geksaedr – Yer, ikosaedr – suv, oktaedr – havo, beshinchi ko'pyoq – dodekaedr esa butun olam tuzilishining belgisi ("beshinchi mohiyat") deb atashgan.

XVIII asrda ko'pyoqlar nazariyasiga Leonard Eyler (1707–1783) salmoqli hissa qo'shdi. 1758- yilda e'lon qilingan qavariq ko'pyoqlarning uchlari, qirralari va yoqlari soni orasidagi muosabat haqidagi Eyler teoremasi va uning isboti rang-barang ko'pyoqlar dunyosiga tartib o'rnatdi va uning go'zal geometrik zojibasini algebraik nuqtayi nazaridan bayon etdi.

Mavzuga oid masalalar va amaliy topshiriqlar

142. Ikki tekislik orasidagi burchak 47°. Bu tekisliklar kesishishidan hosil bo'lgan ikkiyoqli burchaklarning gradus o'lchovini toping.

143. Ikkiyoqli burchakning gradus o'lchovi 52° ga teng. Bu burchakka qo'shni bo'lgan ikkiyoqli burchakning gradus o'lchovi nimaga teng bo'ladi?

144. Tekis burchagi 100° bo'lgan ikkiyoqli burchakning yoqlariga perpendikular bo'lgan to'g'ri chiziqlar orasidagi burchakni toping.

145. Qo'shni ikkiyoqli burchaklarning bissektorlari orasidagi ikkiyoqli burchakning gradus o'lchovi nimaga teng?

146. *A* nuqta gradus o'lchovi 60° bo'lgan ikkiyoqli burchakning bissektorida yotibdi. Agar bu nuqta ikkiyoqli burchak qirrasidan 10 cm masofada yotgan bo'lsa, unda ikkiyoqli burchakning yoqlarigacha bo'lgan masofalarni toping.

147. *A* nuqta gradus o'lchovi 30° bo'lgan ikkiyoqli burchakning bitta yog'iga tegishli bo'lib, ikkinchi yog'idan 6 cm masofada yotibdi. Bu nuqtadan ikkiyoqli burchakning qirrasigacha bo'lgan masofani toping.

148*. *A* nuqta to'g'ri ikkiyoqli burchakning yoqlaridan 3 dm va 4 dm masofada yotibdi. Bu nuqtadan ikkiyoqli burchakning qirrasigacha bo'lgan masofani toping.

149*. Muntazam tetraedrning barcha ikkiyoqli burchaklari teng ekanligini isbotlang va ularning gradus o'lchovini toping.

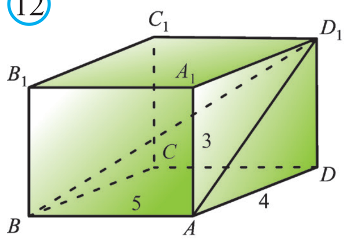
150. Tekis burchaklari: a) 30°; 60°; 20°; b) 45°; 80°; 130°; c) 30°; 60°; 20°;

d) 20°; 60°; 70°; e) 76°; 34°; 110° bo‘lgan uch yoqli burchak mavjudmi?

151*. Qavariq ko‘pyoqli burchakning barcha tekis burchaklari yig‘indisi 360° dan kichik ekanligini isbotlang.

152. To‘g‘ri burchakli parallelepipedda $AB=5$, $AD=4$ va $AA_1=3$ bo‘lsa, ABD_1 burchakni toping (12- rasm).

12



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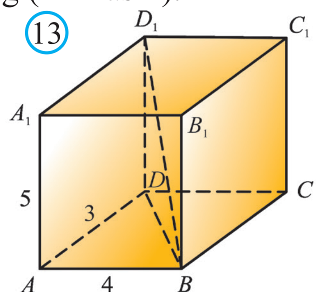

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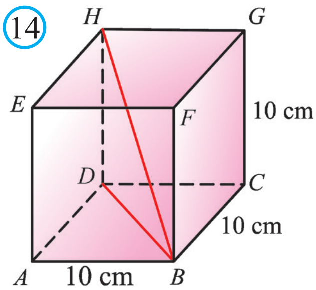


153. To‘g‘ri burchakli parallelepipedda $AB=4$, $AD=3$ va $AA_1=5$ bo‘lsa, DBD_1 burchakni toping (13- rasm).

154. 14- rasmda berilgan kubdagi *DBH* burchakni toping.

155*. *n* ta uchi bor qavariq ko‘pyoqning barcha tekis burchaklari yig‘indisi $360°(n-2)$ ga teng ekanligini isbotlang.

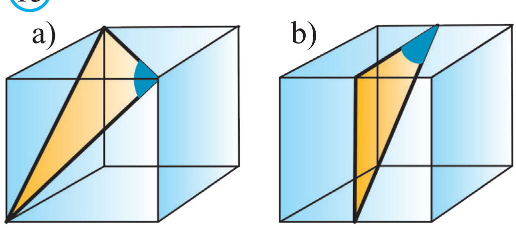
156*. Ko‘pyoq tekis burchaklarining soni uning qirralari sonidan ikki marta ko‘p bo‘lishini isbotlang.

157*. Ko‘pyoq tekis burchaklari soni har doim juft bo‘lishini isbotlang.

158*. Ko‘pyoqning tekis burchaklari yig‘indisi $360°(Y-Q)$ ga teng bo‘lishini isbotlang..

159. 15- rasmlardagi kublarda ajratib ko‘rsatilgan burchaklar kattaligini aniqlang.

15



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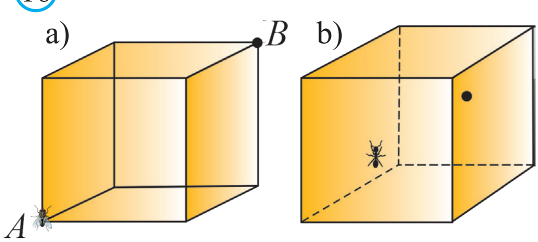


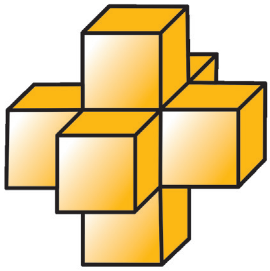
160*. 16- rasmlardagi kubning sirtidagi pashshaga: a) *A* uchdan *B* uchga; b) kub yog‘ining markazidan qarama-qarshi yog‘ining markaziga olib boradigan eng qisqa yo‘lni ko‘rsating (ko‘rsatma: kubning yoyilmasidan foydalaning).

161. 17- rasmda tasvirlangan fazoviy shakl muntazam ko‘pyoqli bo‘ladimi? Uning sirti nechta kvadratdan iborat? Uning nechta uchi va qirrasi bor?

162. 18- rasmda tasvirlangan yoyilmalarning qaysi biri oktaedrga tegishli?

163. 19- rasmda tasvirlangan, kubga ichki chizilgan ko‘pyoqning: a) muntazam tetraedr; b) oktaedr ekanligini asoslang.

17



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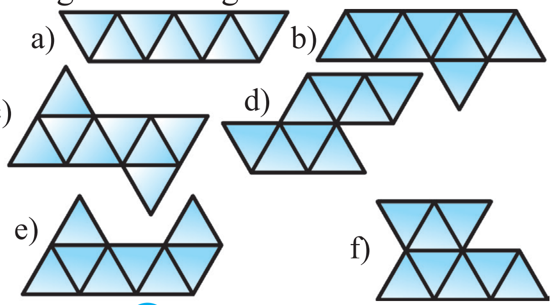

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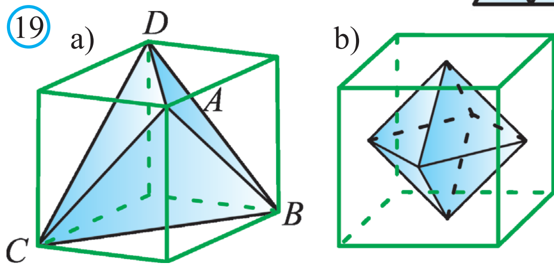

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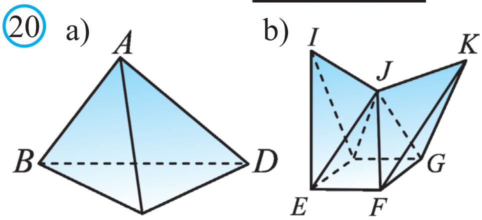


164. 20- rasmda tasvirlangan ko‘pyoqlarning uchlari, qirralari va yoqlari sonini aniqlab, ularni Eyler tenglamasiga qo‘yib tekshiring.

165. Qavariq ko‘pyoqning har bir uchidan uchtadan qirra chiqadi. Agar bu ko‘pyoqning qirralar soni: a) 12; b) 15 ga teng bo‘lsa, uning nechta uchi va yog‘i bor?

166*. 13 ta yog‘i va har bir yog‘ida 13 tadan qirrasi bo‘lgan ko‘pyoq mavjudmi?

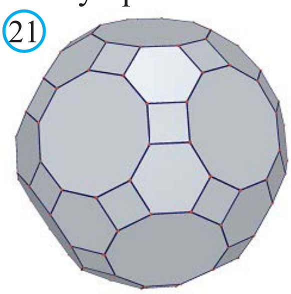
167. Qavariq ko‘pyoqning har bir uchidan to‘rttadan qirra chiqadi. Agar bu ko‘pyoqning qirralar soni 12 ga teng bo‘lsa, uning nechta uchi va yog‘i bor?

168. a) Muntazam tetraedr; b) kub; c) oktaedr; d) dodekaedr; e) ikosaedrning uchlari, qirralari va yoqlari sonini toping va bu ko‘pyoqlar uchun Eyler tenglamasining o‘rinli bo‘lishini tekshiring.

169. Uchlari soni 8 ta, qirralari soni esa 12 ta bo‘lgan muntazam ko‘pyoqning yoqlari sonini toping va uning nomini aniqlang.

170. Uchlari soni 6 ta, qirralari soni esa 12 ta bo‘lgan muntazam ko‘pyoqning yoqlari sonini toping va uning nomini aniqlang.

21



171. Uchlari soni 10 ta, yoqlari soni esa 7 ta bo‘lgan ko‘pyoqning qirralari sonini toping.

172. Uchlari soni 14 ta, qirralari soni esa 21 ta bo‘lgan ko‘pyoqning yoqlari sonini toping.

173. 21- rasmdagi ko‘pyoqning 62 ta yog‘i va 120 ta uchi bor bo‘lsa, uning qirralari sonini toping.

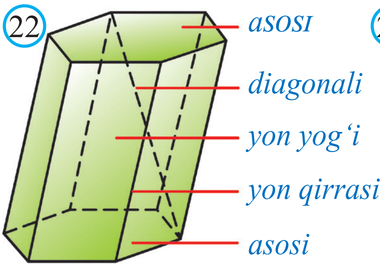
6. PRIZMA VA UNING SIRTI

6.1. Prizma va uning kesimlari

Prizmalar bilan quyi sinflardan tanishsiz. Shunday bo‘lsada, ularga oid ba’zi tushuncha va xossalarni eslatib o‘tamiz.

Prizma deb ikki yog‘i (asosi) teng *n* burchakdan, qolgan *n* ta yoqlari esa parallelogrammlardan iborat ko‘pyoqqa aytiladi (22- rasm).

22



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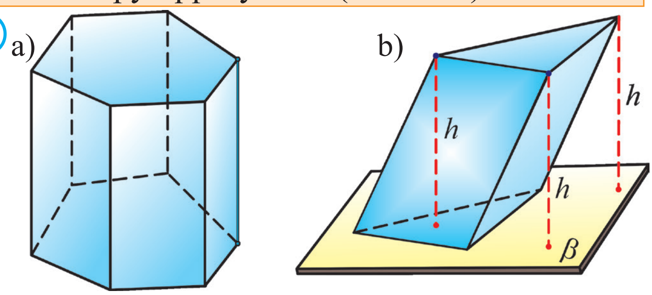
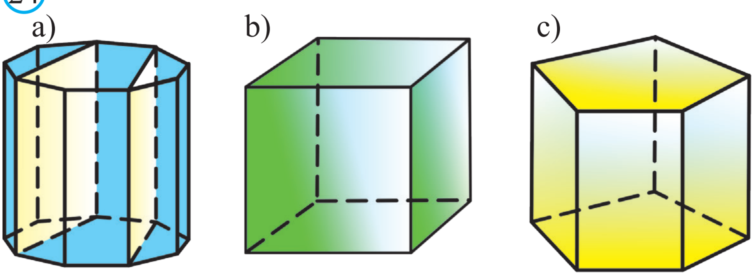


Prizma yon yoqlarining asosiga perpendikular yoki perpendikular emasligiga qarab *to‘g‘ri* yoki *og‘ma* prizmalarga ajratiladi. 23.a- rasmda to‘g‘ri oltiburchakli prizma, 23.b- rasmda esa og‘ma uchburchakli prizma tasvirlangan. Ravshanki, to‘g‘ri prizmaning yon yoqlari to‘g‘ri to‘rtburchaklardan iborat bo‘ladi.

24



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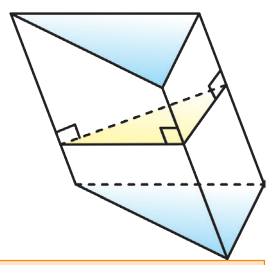


Asosi muntazam ko‘pburchakdan iborat to‘g‘ri prizma *muntazam prizma* deb ataladi (24- rasm). Muntazam prizmaning yon yoqlari bir-biriga teng to‘g‘ri to‘rtburchaklardan iborat bo‘ladi.

Prizma asosining biror nuqtasidan ikkinchi asosiga tushirilgan perpendikular prizmaning *balandligi* deb ataladi (23.b- rasm).

Prizmaning *diagonal kesimi* deb, prizma asoslarining mos diagonallari orqali o‘tkazilgan kesimga aytiladi (24.a- rasm). Prizma diagonal kesimlarining soni prizma bitta asosining diagonallari soniga teng.

Prizmaning *perpendikular kesimi* deb, uning barcha yon qirralariga perpendikular kesimga aytiladi (25- rasm).

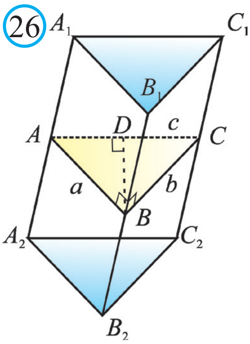
Qavariq n-burchakning $\frac{n(n-3)}{2}$ ta diagonali borligini hisobga olsak, n-burchakli prizma diagonal kesimlari soni ham $\frac{n(n-3)}{2}$ ta bo‘ladi.

Har bir diagonal kesimda prizmaning ikkita diagonalini o‘tkazish mumkin. Demak, n-burchakli prizmaning jami $n(n-3)$ ta diagonali bor.

1- masala. Uchburchakli og‘ma prizma yon qirralari orasidagi masofalar, mos ravishda, 7 cm, 15 cm va 20 cm. Prizmaning eng katta yuzli yon yog‘idan uning qarshisidagi yon qirrasigacha bo‘lgan masofani toping.

Yechish. Ma’lumki, parallel to‘g‘ri chiziqlar orasidagi masofa bu to‘g‘ri chiziqlar birining biror nuqtasidan ikkinchisiga o‘tkazilgan perpendikularning uzunligiga teng. Unda berilgan prizmaning ABC perpendikular kesimi tomonlarining uzunligi shu masofalarga teng bo‘ladi (26- rasm). Prizmaning eng katta yuzli yog‘ida eng katta $AC=20$ cm tomon yotadi. B_2B_1 qirradan $A_2A_1C_1C_2$ tekislikkacha bo‘lgan masofa ABC uchburchakning BD balandligiga teng bo‘ladi. Unda Geron formulasiga ko‘ra:

26



$$S_{ABC} = \sqrt{p(p-a)(p-b)(p-c)}, \quad p = \frac{a+b+c}{2},$$

$$p = \frac{a+b+c}{2} = \frac{7+15+20}{2} = 21,$$

$$S_{ABC} = \sqrt{21(21-7)(21-15)(21-20)} = \sqrt{21\cdot 14\cdot 6\cdot 1} = 42.$$

Ikkinchi tomondan, $S_{ABC} = \frac{AC\cdot BD}{2}$.

Bundan, $42 = \frac{AC\cdot BD}{2}$ yoki $BD = 4{,}2$ cm.

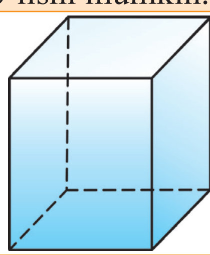
Javob: 4,2 cm.

6.2. Parallelepiped va kub

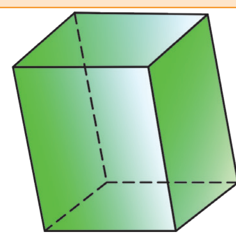
Asoslari parallelogrammdan iborat prizma *parallelepiped* deb ataladi (27- rasm). Parallelepipedlar ham prizma kabi to‘g‘ri (27.a- rasm) va og‘ma (27.b- rasm) bo‘lishi mumkin.

27

a)



b)



Parallelepipedning umumiy uchga ega bo‘lmagan yoqlari *qarama-qarshi yoqlari* deb ataladi.

Parallelepipedning

– 12 ta qirralari bo‘lib, ularning har to‘rttasi teng kesmalardan iborat (28.a- rasm),

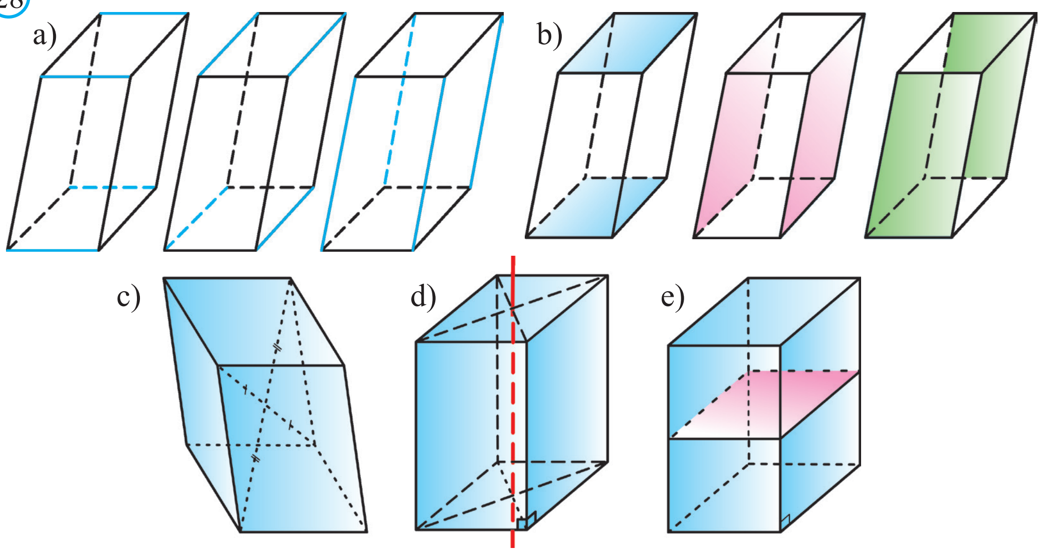
– 6 ta yoqlari bo‘lib, uning qarama-qarshi yoqlari o‘zaro parallel va teng bo‘ladi (28.b- rasm),

– 4 ta diagonali bo‘lib, ular bitta nuqtada keshishadi va kesishish nuqtasida teng ikkiga bo‘linadi (28.c- rasm),

– diagonallari keshishish nuqtasi uning simmetiya markazi bo‘ladi (28.c- rasm).

To‘g‘ri parallelepipedning simmetriya o‘qi (28.d- rasm) va simmetriya tekisligi bor (28.e -rasm).

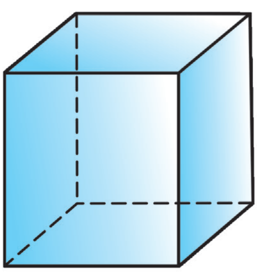
28



Asoslari to‘g‘ri to‘rtburchakdan iborat to‘g‘ri parallelepiped *to‘g‘ri burchakli parallelepiped* deb ataladi (29- rasm).

Ravshanki, to‘g‘ri burchakli parallelepipedning barcha yoqlari to‘g‘ri to‘rtburchaklardan iborat bo‘ladi.

29



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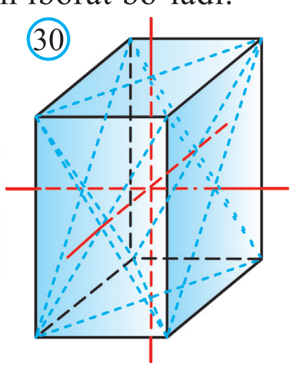

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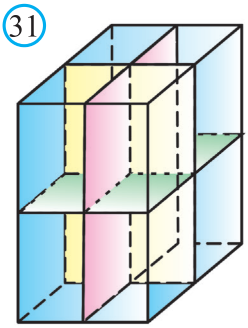
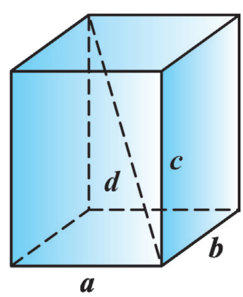


32

$d^2 = a^2 + b^2 + c^2$



To‘g‘ri burchakli parallelepipedning uchta simmetriya o‘qi (30- rasm) va uchta simmetriya tekisligi bor (31- rasm).

To‘g‘ri burchakli parallelepipedning bitta uchidan chiquvchi uchta qirrasi uzunliklariga uning *o‘lchamlari* deb aytiladi.

Xossa: To‘g‘ri burchakli parallelepiped *d* diagonalining kvadrati uning o‘lchamlari: *a, b* va *c* ning kvadratlari yig‘indisiga teng (32- rasm):

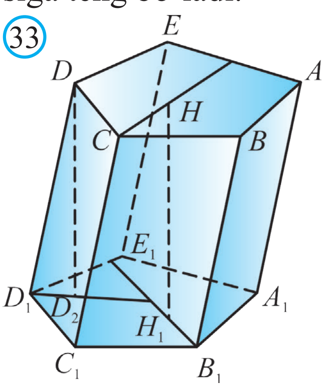
$$d^2 = a^2 + b^2 + c^2.$$

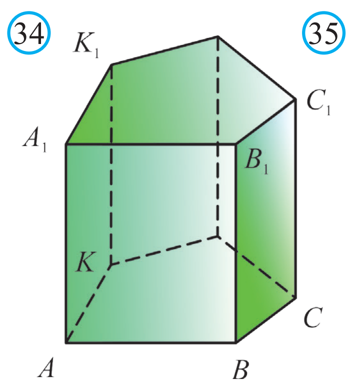
O‘lchamlari teng bo‘lgan to‘g‘ri burchakli parallelepiped *kub* deb ataladi. Ravshanki, kubning barcha yoqlari teng kvadratlardan iborat bo‘ladi. Kub bitta simmetriya markaziga, 9 ta simmetriya o‘qiga va 9 ta simmetriya tekisligiga ega.

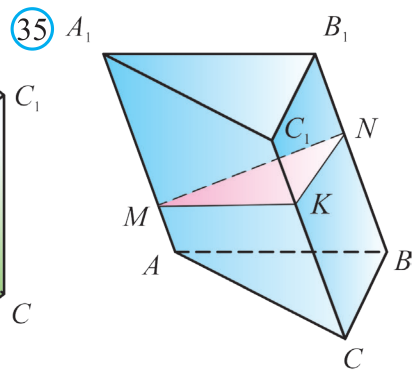
Yuqorida prizmalarning qator xossalarini sanab o‘tdik. Ularning ba’zilarini 10- sinfda isbotlagan edik. Qolgan xossalarni isboti nisbatan sodda bo‘lganligi uchun ularni mustaqil isbotlash uchun qoldirdik.

6.3. Prizmaning yon va to‘la sirti

33- rasmda $ABCDEA_1B_1C_1D_1E_1$ prizmaning HH_1 va DD_1 balandliklari tasvirlangan. Ravshanki, muntazam prizmaning balandligi uning yon qirrasiga teng bo‘ladi.







Prizma *yon sirti* (aniqrog‘i, *yon sirtining yuzi*) uning yon yoqlari yuzi yig‘indisiga teng, *to‘la sirti* esa yon sirti va ikkita asosining yuzi yig‘indisiga teng.

$$S_{to'la} = S_{yon} + 2S_{asos}.$$

Teorema. To‘g‘ri prizmaning yon sirti asosining perimetri bilan balandligining ko‘paytmasiga teng:

$$S_{yon} = P_{asos} \cdot h.$$

Isbot. Berilgan prizmaning balandligi *h*, asosining perimetri $P = AB + BC + \ldots + KA$ bo‘lsin (34- rasm). Ravshanki, to‘g‘ri prizmaning har bir yog‘i to‘g‘ri to‘rtburchakdan iborat. Bu to‘g‘ri to‘rtburchaklarning asosi prizmaning mos tomonlariga, balandligi esa prizma balandligiga teng.

Demak, $S_{yon}=AB \cdot h+ BC \cdot h+...+ KA \cdot h=(AB+ BC + ... KA)\cdot h = P\cdot h.$ □

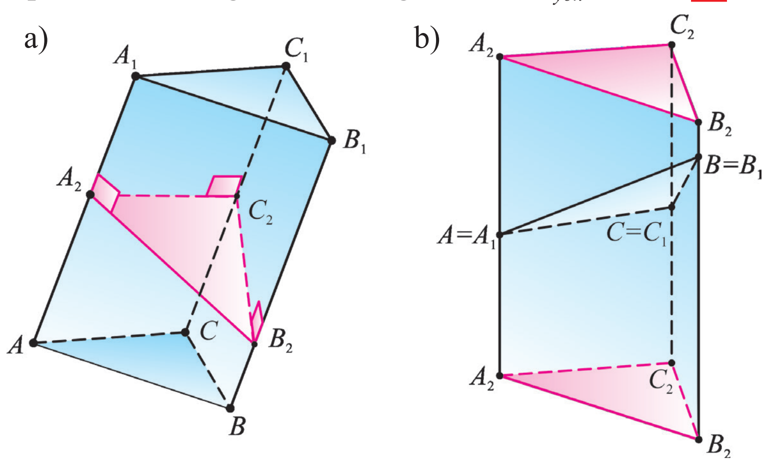
Teorema. Ihtiyority prizmaning yon sirti uning perpendikular kesimi perimetri bilan yon qirrasi uzunligining ko‘paytmasiga teng:

$$\boldsymbol{S_{yon} = P \cdot l.}$$

Isbot. Perpendikular kesimning perimetri P ga teng bo‘lsin (35- rasm). Kesim prizmani ikki bo‘lakka ajratadi (36.a- rasm). Bu bo‘laklarning birini olib, prizma asoslari ustma-ust tushadigan qilib parallel ko‘chiramiz. Natijada yangi to‘g‘ri prizma hosil bo‘ladi (36.b- rasm). Ravshanki, bu prizmaning yon sirti berilgan prizma yon sirtiga teng. Uning asosi berilgan perpendikular kesimdan iborat bo‘lib, yon qirrasi l ga teng bo‘ladi.

Demak, yuqorida isbotlangan teoremaga ko‘ra: $S_{yon} = P \cdot l$ □

36



Mavzuga oid masalalar va amaliy topshiriqlar

174. Tetraedr bitta yog‘ining yuzi 6 cm^2 bo‘lsa, uning to‘la sirtini toping.

175. Oktaedr bitta yog‘ining yuzi 5,5 cm^2 bo‘lsa, uning to‘la sirtini toping.

176. Dodekaedr bitta yog‘ining yuzi 6,4 cm^2 bo‘lsa, uning to‘la sirtini toping.

177. Kub to‘la sirtining yuzi 105,84 cm^2 bo‘lsa, uning har bir yog‘i yuzini va qirrasining uzunligini toping.

178. Oktaedr to‘la sirtining yuzi $32\sqrt{3}$ cm^2 bo‘lsa, uning har bir yog‘i yuzini va qirrasining uzunligini toping.

179. To‘g‘riburchakli parallelepiped asosining tomonlari 7:24 nisbatda, diagonal kesimining yuzi 50 dm^2 ga teng. Yon sirtining yuzini toping.

180*. To‘g‘ri parallelepipedning yon qirrasi 1 m ga, asoslarining tomonlari

23 m va 11 m ga teng. Asos diagonallarining nisbati 2:3 kabi. Diagonal kesimlarining yuzini toping.

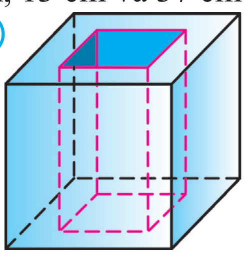
181. To‘g‘ri parallelepiped asosining tomonlari 3 cm va 5 cm, asosining diagonallaridan biri 4 cm ga teng. Parallelepiped kichik diagonallaridan biri asos tekisligi bilan 60° li burchak tashkil etadi. Uning diagonallari uzunligini toping.

182. To‘g‘ri parallelepipedning yon qirrasi 5 m, asosining tomonlari 6 m va 8 m, asosining diagonallaridan biri 12 m ga teng. Parallelepipedninng diagonallarini toping.

183*. Uchburchakli muntazam prizmaning qirrasi 3 ga teng. Asosining tomoni va o‘qining o‘rtasi orqali tekislik o‘tkazilgan. Kesimning yuzini toping.

184. Uchburchakli to‘g‘ri prizmaning balandligi 50 cm, asosining tomonlari 40 cm, 13 cm va 37 cm. Prizmaning to‘la sirtini toping.

37



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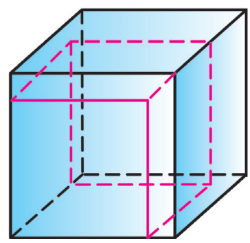

185*. 37- rasmda tasvirlangan birlik kubdan asosining tomoni 0,5 ga yon qirrasi 1 ga teng bo‘lgan muntazam to‘rtburchakli prizma o‘yib olindi. Kubning qolgan bo‘lagi to‘la sirtining yuzini hisoblang.

186. Agar kubning qirrasi 1 birlik orttirilsa, uning to‘la sirti 54 birlikka ortadi. Kubning qirrasini toping (38- rasm).

187. $ABCC_1B_1A_1$ og‘ma prizmaning asosi ABC tengyonli uchburchak bo‘lib, unda $AB=AC=10$ cm va $BC=12$ cm. A_1 uchi A, B va C uchlardan teng uzoqlikda yotadi hamda AA_1 kesma 13 cm ga teng. Prizmaning to‘la sirtini toping.

188. Muntazam to‘rtburchakli prizmaning yon sirti 160 ga, to‘la sirti 210 ga teng. Prizma asosining diagonalini toping.

189. Uchburchakli og‘ma prizmaning yon qirralari yotgan parallel to‘g‘ri chiziqlar orasidagi masofa 2 cm , 3 cm va 4 cm, yon qirralari esa 5 cm ga teng. Prizmaning yon sirtini toping.

190. Kubning qirralari uzunliklari yig‘indisi 96 ga teng. Uning yon sirtini toping.

191. 39- rasmlarda tasvirlangan ko'pyoqlarning to'la sirtini hisoblang (hamma ikkiyoqli burchaklar to'g'ri burchak).

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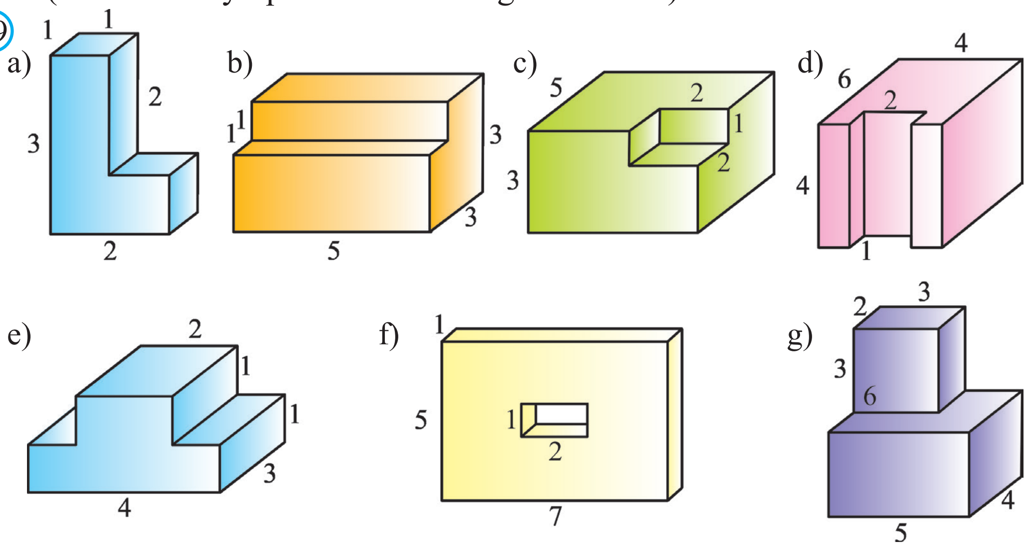

192. 40- rasmlarda tasvirlangan ko'pyoqlarning to'la sirtini hisoblang (hamma ikkiyoqli burchaklar to'g'ri burchak).

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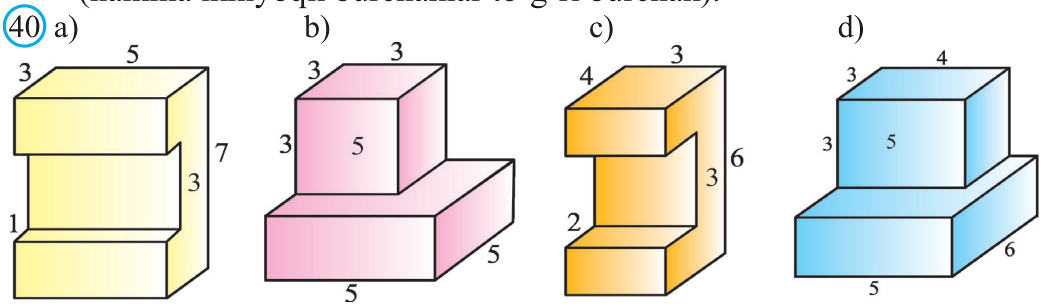

193. Oltiburchakli muntazam prizmaning yon qirrasi 8 cm, asosining tomoni esa 3 cm. Prizmaning barcha qirralari uzunliklarining yig'indisini toping.

194. To'rtburchakli muntazam prizma asosining tomoni 6 cm, prizmaning balandligi esa 5 cm. Uning diagonal kesimi yuzini toping.

195. Uchburchakli muntazam prizma asosining tomoni 6 cm, yon qirrasi esa 12 cm. Prizma yon sirtining yuzini toping.

196. 41- rasmlarda tasvirlangan ko'pyoqlarning to'la sirtini hisoblang (hamma ikkiyoqli burchaklar to'g'ri burchak).

197. 42- rasmlarda tasvirlangan ko'pyoqlarning to'la sirtini hisoblang (hamma ikkiyoqli burchaklar to'g'ri burchak).

198*. 43- rasmdadagi uy asosining o'lchamlari 6 m va 8 m. Uyning tomi asosiga 45° li burchak ostida og'gan. Tom sirti yuzini toping.

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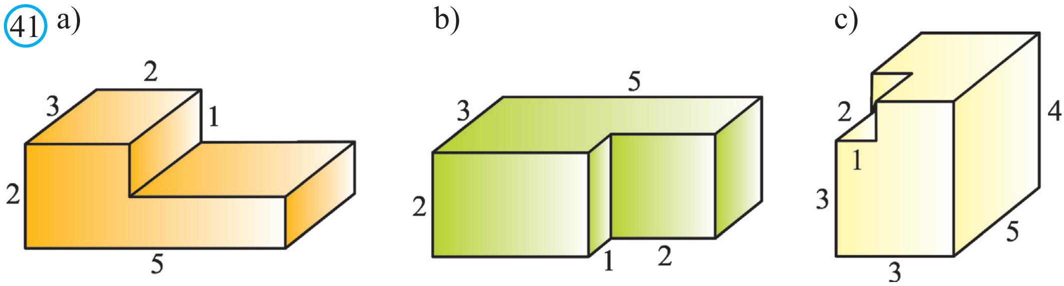

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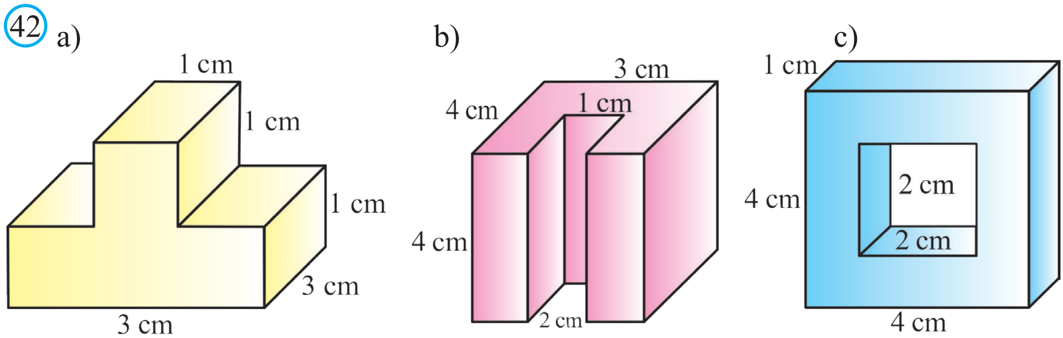

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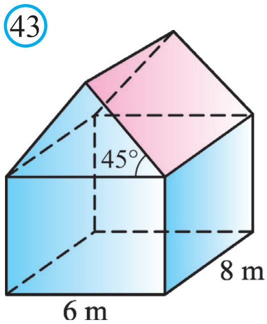

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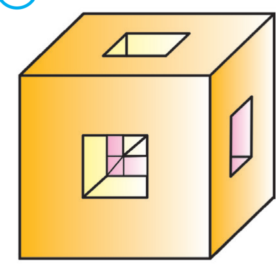

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199. Parallelepipedning bitta uchidan chiquvchi qirralari, mos ravishda, 6 cm, 8 cm va 12 cm. Parallelepiped barcha qirralari uzunliklarining yig‘indisini toping.

200. Parallelepipedning bitta umumiy uchga ega yoqlarining yuzlari 6 cm^2, 12 cm^2 va 16 cm^2. Parallelepiped to‘la sirtining yuzini toping.

201*.Qirrasi 3 cm ga teng bo‘lgan kubning har bir yog‘idan ko‘ndalang kesimi - asosi 1 cm ga teng kvadrat shaklidagi teshiklar o‘yilgan (44-rasm). Kubning qolgan qismi to‘la sirtining yuzini toping.

202*.Futbol to‘pining sirti qirralari 5 cm ga teng bo‘lgan 12 ta muntazam beshburchak va 20 ta muntazam oltiburchakdan iborat (45- rasm). Futbol to‘pining to‘la sirtini toping. To‘p kvadrat santimetri 60 so‘m turadigan charmdan ishlangan va uning 10 foizi chok va chiqitga chiqishi ma’lum bo‘lsa, to‘pga sarflangan charm narxini toping.

7. PRIZMANING HAJMI

7.1. Hajm tushunchasi

Fazoda geometrik jismga xos bo‘lgan xususiyatlardan biri bu hajm tushunchasidir. Har qanday predmet (jism) fazoning qandaydir qismini egallaydi. Masalan, g‘isht gugurt qutisiga qaraganda kattaroq joyni egallaydi. Bu qismlarni o‘zaro taqqoslash uchun hajm tushunchasi kiritiladi.

Hajm – fazoviy jismning quyidagi xossalarga ega bo‘lgan miqdoriy (sonli) ko‘rsatkichidir:

1. Har qanday jism musbat sonlarda ifodalanuvchi muayyan hajmga ega.

2. Teng jismlar hajmi ham teng.

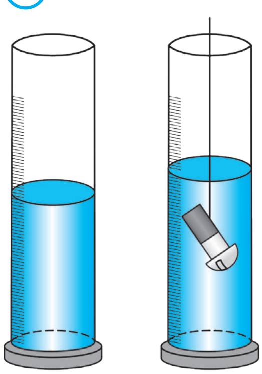
3. Agar jism bir necha bo‘lakka bo‘lingan bo‘lsa, uning hajmi bo‘laklar hajmlari yig‘indisiga teng.

4. Qirrasi bir birlik uzunlikka teng kubning hajmi birga teng.

Hajm – uzunlik va yuz kabi sonli kattaliklardan biridir. Uzunlik o‘lchov birligining tanlanishiga qarab *birlik* (qirrasi birlik uzunlikka ega) *kub* ning hajmi 1 cm^3, 1 dm^3, 1 m^3 va hokazo hajm birliklari bilan o‘lchanadi.

Jismlar hajmini turli usullar bilan o‘lchashadi yoki hisoblashadi. Masalan, kichikroq detalning hajmini bo‘linmalarga (shkalaga) ega bo‘lgan idish (menzurka) yordamida o‘lchash mumkin (46- rasm). Chelak hajmini esa unga birlik hajmga ega bo‘lgan idish yordamida suv quyib, to‘ldirish bilan o‘lchash mumkin (47- rasm). Lekin hamma jismlarning ham hajmni bunday usullar bilan o‘lchab bo‘lmaydi. Bunday hollarda hajm turli usullar bilan hisoblanadi. Quyida shu usullar xususida to‘xtalamiz va ularning ba’zilarini isbotsiz keltiramiz.

46



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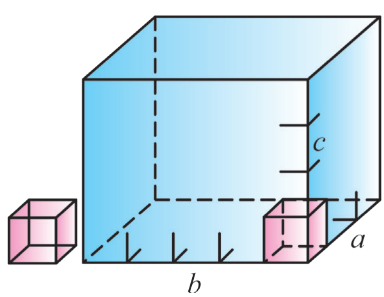

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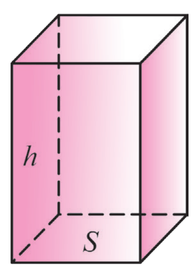


7.2. Parallelepipedning hajmi

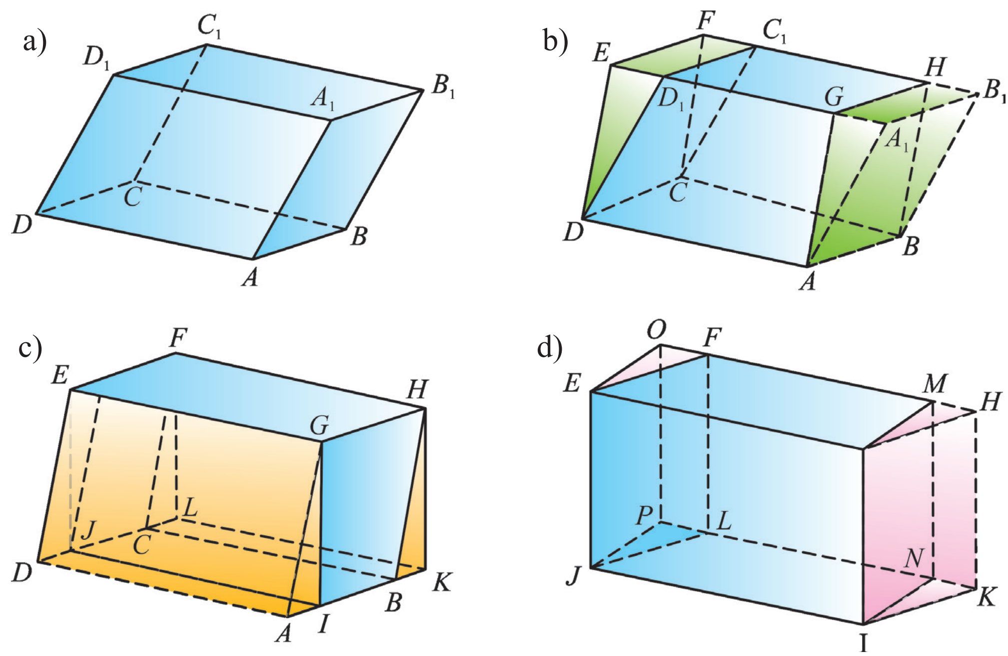
Teorema. To‘g‘ri burchakli parallelepipedning hajmi uning uchta o‘lchamlari ko‘paytmasiga teng (48- rasm): $V = a \cdot b \cdot c.$

Natija. To‘g‘ri burchakli parallelepipedning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng (49- rasm): $V = S \cdot h.$

Teorema. Ixtiyoriy parallelepipedning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng (50- rasm): $V = S \cdot h.$

Mazkur xossa yuqoridagi natijadan kelib chiqadi. Quyidagi 50- rasmlarda berilgan parallelepiped qanday qilib to‘g‘ri burchakli parallelepipedga to‘ldirilishi tasvirlangan. Bundan foydalanib xossani mustaqil asoslang.

50



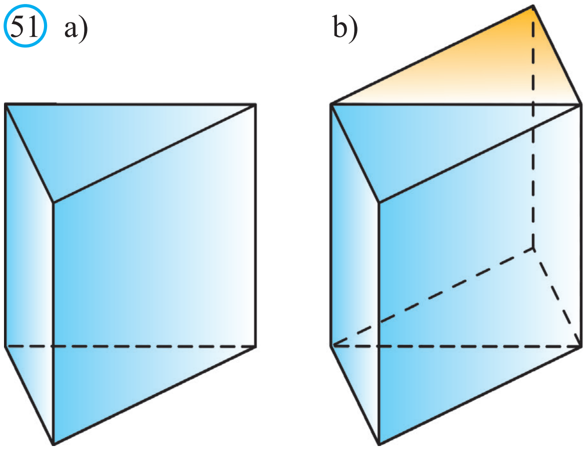
7.3. Prizmaning hajmi

Teorema. To‘g‘ri prizmaning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng (51- rasm): $V = S \cdot h.$

Isbot. 1- hol. Asosi to‘g‘ri burchakli uchburchakdan iborat to‘g‘ri prizma berilgan bo‘lsin (51.a- rasm). Bu prizmani unga teng bo‘lgan prizma bilan to‘g‘ri burchakli paralelepipedgacha to‘ldirish mumkin (51.b- rasm).

Berilgan prizmaning hajmi, asosining yuzi va balandligi, mos ravishda, V, S va h bo‘lsa, hosil bo‘lgan to‘g‘ri burchakli parallelepipedning hajmi, asosining yuzi va balandligi, mos ravishda, $2V$, $2S$ va h bo‘ladi.

51 a) b)



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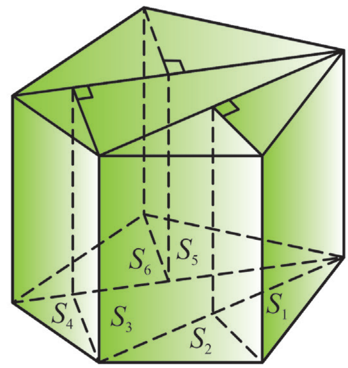


Demak, $2V=2S\cdot h$ yoki $V=S\cdot h$ bo'ladi.

2- hol. Ixtiyoriy to'g'ri n-burchakli prizma berilgan bo'lib, uning asosi yuzi S, balandligi esa h ga teng bo'lsin. Prizmaning asosi – n-burchakni uning diagonallari bilan uchburchaklarga, uchburchaklarning har birini esa to'g'ri burchakli uchburchaklarga bo'lish mumkin (52- rasm). Natijada, berilgan prizmani chekli sondagi asosi to'g'ri burchakli uchburchaklardan iborat to'g'ri prizmalarga ajratish mumkinligini aniqlaymiz. Bu prizmalarning balandligi h ga teng bo'lib, ularning asoslari yig'indisi berilgan prizma yuziga teng bo'ladi: $S = S_1+ S_2+...+S_k$.

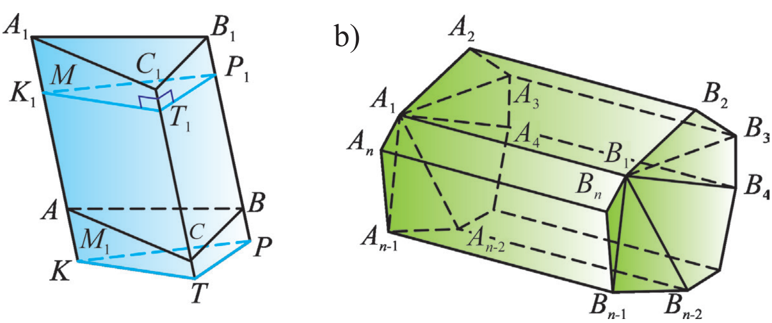
Berilgan prizmaning hajmi uni tashkil qiluvchi uchburchakli prizmalar hajmlari yig'indisidan iborat bo'ladi:

$$V = S_1h + S_2h +...+S_kh=(S_1+S_2+...+S_k)\,h = S\cdot h, \quad \text{yoki } V = S\cdot h. \square$$

Teorema. Ixtiyoriy prizmaning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng: $\boldsymbol{V = S\cdot h}$.

Bu teoremani 5.3- rasmdan foydalanib, oldin uchburchakli prizma uchun (5.3.a- rasm), so'ng ixtiyoriy prizma uchun (5.3.b- rasm) mustaqil isbotlang.

53 a) b)



1- masala. To'g'ri parallelepiped asosining tomonlari a va b ga teng bo'lib, ular o'zaro 30° li burchak tashkil qiladi. Agar parallelepipedning yon sirti S ga teng bo'lsa, uning hajmini toping.

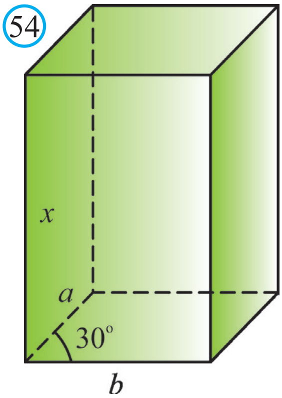
Yechish: Parallelepiped balandligini h bilan belgilaymiz (54- rasm). Unda shartga ko'ra:

$$S = (2a+2b)\,h \quad \text{yoki} \quad h = \frac{S}{2(a+b)}.$$

$$S_{asos} = ab\sin 30^\circ = \frac{ab}{2}.$$

$$V = S_{asos}\cdot h = \frac{ab}{2}\cdot\frac{S}{2(a+b)} = \frac{abS}{4(a+b)}.$$

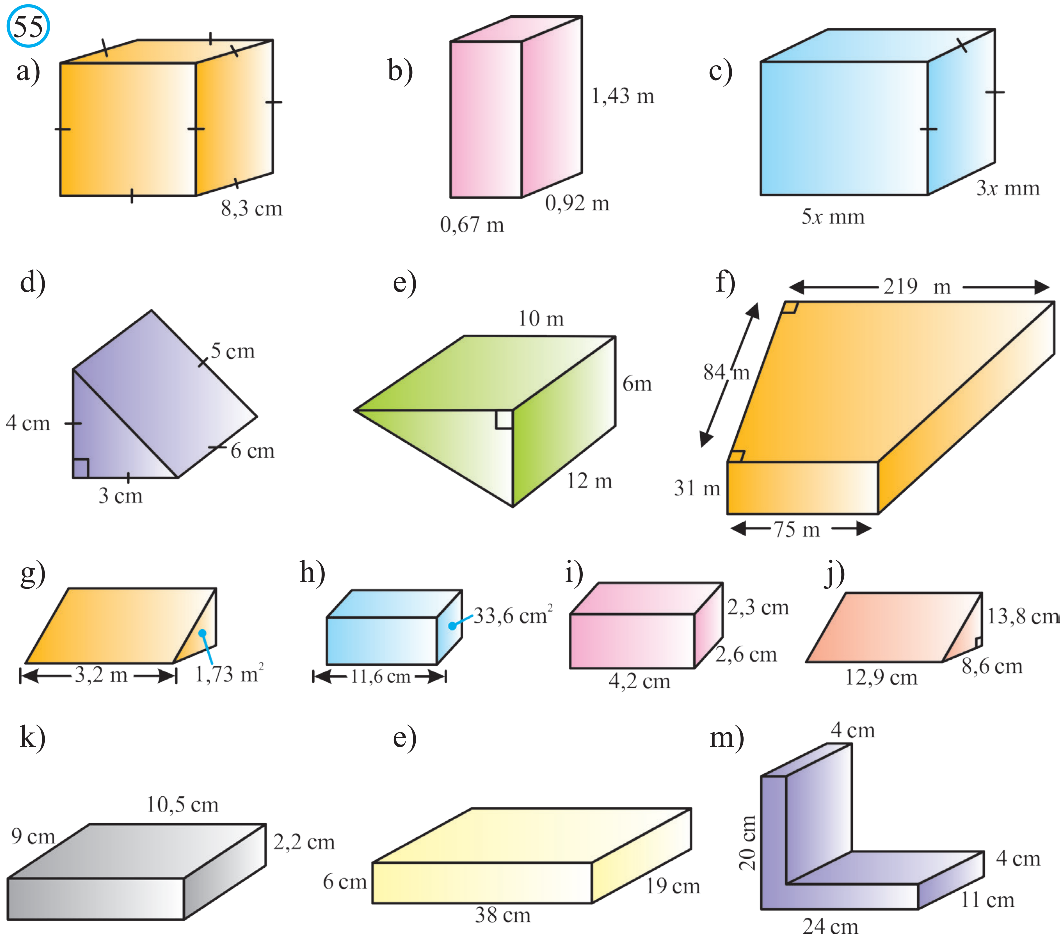
54



Mavzuga oid masalalar va amaliy topshiriqlar

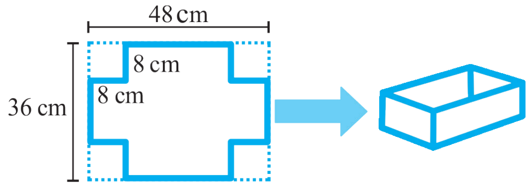
203. 55- rasmda tasvirlangan ko'pyoqlarning hajmini toping.

55



204. 56- rasmda berilgan yoyilmaga ko'ra yasalgan idishning hajmini toping.

56



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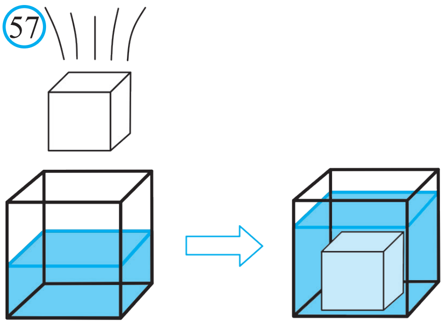

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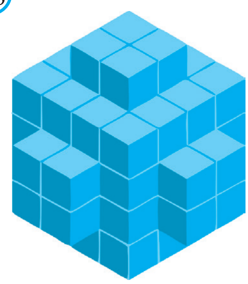


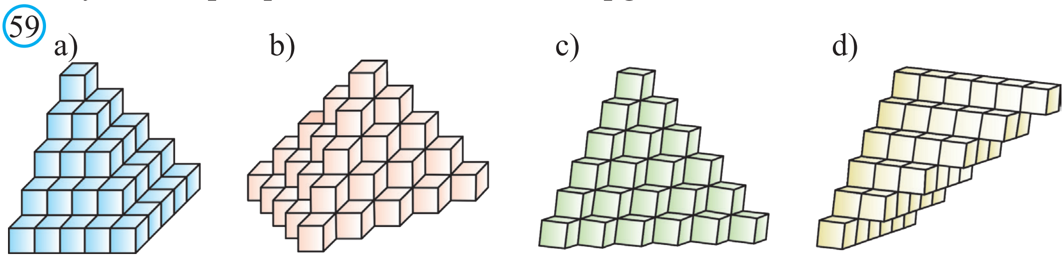
205*. 57- rasmga ko‘ra masala tuzing va uni yeching.

206. 58- rasmda keltirilgan jism 88 ta birlik kubchadan yasalgan. Jismning to‘la sirtini toping.

207. To‘g‘ri burchakli parallelepiped yog‘ining yuzi 12 ga va unga perpendikular qirra uzunligi 12 ga teng. Parallelepipedning hajmini toping.

208. 59- rasmda tasvirlangan fazoviy shakllardan qaysi birining hajmi katta, ya’ni ko‘proq kubchalardan tashkil topgan?

59



209. To‘g‘ri burchakli parallelepiped hajmi 24 ga teng va qirralaridan birining uzunligi 3 ga teng. Parallelepipedning bu qirraga perpendikular yog‘ining yuzini toping.

210. To‘g‘ri burchakli parallelepiped hajmi 60 ga teng va yoqlaridan birining yuzi 12 ga teng. Parallelepipedning bu yoqqa perpendikular qirraning uzunligini toping.

211. To‘g‘ri burchakli parallelepiped bir uchidan chiquvchi uchta qirralari uzunliklari 4, 6 va 9 ga teng. Unga tengdosh kub qirrasini toping.

212. Kubning to‘la sirti yuzi 18 ga teng bo‘lsa, uning diagonalini toping.

213. Kubning hajmi 8 ga teng bo‘lsa, uning to‘la sirtining yuzini toping.

214. Agar kubning qirralarini 1 birlik orttirilsa, uning hajmi 19 birlikka ortadi. Kubning qirrasini toping.

215. Kubning to‘la sirtining yuzi 24 ga teng. Uning hajmini toping.

216. Kubning diagonali $\sqrt{12}$ ga teng bo‘lsa, uning hajmini toping.

217. Kubning hajmi $24\sqrt{3}$ ga teng bo‘lsa, uning diagonalini toping.

218. Birinchi kubning hajmi ikkinchisinikidan 8 marta katta. Birinchi kubning to‘la sirtining yuzi ikkinchisinikidan necha marta katta?

219. Qirrasi 30 cm bo‘lgan kub shaklidagi idishga (sisternaga) necha litr suv ketadi?

220. To‘g‘ri burchakli parallelepipedning bitta uchidan chiquvchi qirralari 2 va 6 ga teng. To‘g‘ri burchakli parallelepiped hajmi 48 ga teng. Parallelepipedning shu uchidan chiquvchi uchinchi qirrasini toping.

221. To‘g‘ri parallelepiped asosining tomonlari uzunligi $2\sqrt{2}$ cm va 5 cm, ular orasidagi burchak 45° ga teng. Agar parallelepipedning kichik diagonali 7 cm ga teng bo‘lsa, uning hajmini toping.

222*. To‘g‘ri parallelepiped asosining a va b tomonlari 30° li burchak tashkil qiladi. To‘la sirti S ga teng. Uning hajmini toping.

223. To‘g‘ri burchakli parallelepipedning o‘lchamlari 15 m, 50 m va 36 m. Unga tengdosh kubning qirrasini toping.

224. Uchburchakli to‘g‘ri prizma asosining tomonlari 29, 25 va 6 ga, qirrasi esa asosining katta balandligiga teng. Prizmaning hajmini toping.

225. 39- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

226. 40- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

227. To‘g‘ri parallelepipedning asosining yuzi 1 m^2 bo‘lgan rombdan iborat. Diagonal kesimlarining yuzi, mos ravishda, 3 m^2 va 6 m^2. Parallelepipedning hajmini toping.

228. 41- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

229. 42- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

230. Kengligi 3 m va uzunligi 20 m bo‘lgan yolakka qalinligi 10 cm bo‘lgan asfalt qatlami yotqizildi. Yo‘lak uchun qancha hajmdagi asfalt ishlatildi?

231*. Og‘ma parallelepipedning asosi – tomoni 1 m ga teng bo‘lgan kvadratdan iborat. Yon qirralaridan biri 2 m ga teng va asosining o‘ziga yopishgan har bir tomoni bilan 60° li burchak tashkil etadi. Parallelepipedning hajmini toping.

232*. Parallelepipedning yoqlari – tomoni a ga teng va o‘tkir burchagi 60° bo‘lgan teng romblardan iborat. Parallelepipedning hajmini toping.

233. Parallelepipedning har bir qirrasi 1 cm ga teng. Parallelepipedning bir uchidagi uchala yassi burchagi o‘tkir bo‘lib, har biri $2a$ ga teng. Parallelepipedning hajmini toping.

234*. Parallelepipedning bir uchidan chiquvchi uchta qirrasining uzunliklari a, b, c ga teng. a va b qirralari o‘zaro perpendikular, c qirra esa ularning har biri bilan α burchak tashkil etadi. Parallelepipedning hajmini toping (60- rasm).

235. a) Uchburchakli; b) to‘rtburchakli; c) oltiburchakli muntazam prizma asosining tomoni a va yon qirrasi b bo‘yicha hajmini toping.

236. To‘g‘ri parallelepiped asosining tomonlari a cm va b cm ga teng bo‘lib, ular o‘zaro α burchak tashkil qiladi. Parallelepipedning kichik diagonali d ga teng bo‘lsa, uning hajmini toping.

237. Uchburchakli og‘ma prizmaning yon qirralari 15 m ga, ular orasidagi masofa esa 26 m, 25 va 17 m ga teng. Prizmaning hajmini toping.

238. To‘rtburchakli muntazam prizmaning diagonali 3,5 cm ga, yon yog‘ining diagonali 2,5 cm ga teng. Prizmaning hajmini toping.

239. Uchburchakli muntazam prizma asosining tomoni a ga, yon sirti asoslari yuzlarining yig‘indisiga teng. Uning hajmini toping.

240. Oltiburchakli muntazam prizmada eng katta diagonal kesimning yuzi 4 m^2 ga, ikkita qarama-qarshi yon qirralari orasidagi masofa 2 m ga teng. Prizmaning hajmini toping.

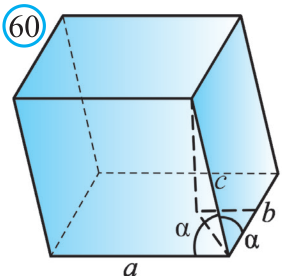
241*. Yetti marta kir yuvishdan keyin sovunning o‘lchamlari ikki marta kamaydi (61- rasm). Agar har kir yuvganda bir xil hajmdagi sovun sarflangani ma’lum bo‘lsa, sovun yana necha marta kir yuvishga yetadi?

242*. Og‘ma prizmada yon qirralariga perpendikular va hamma yon qirralarini kesib o‘tadigan tekislik o‘tkazilgan. Hosil qilingan kesim yuzi Q, yon qirralari esa l ga teng bo‘lsa, prizmaning hajmini toping (62- rasm).

243. Uchburchakli to‘g‘ri prizma asosining tomonlari 4 cm, 5 cm,7 cm ga, yon qirrasi esa asosining katta balandligiga teng. Prizmaning hajmini toping.

244. 63- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang.

60



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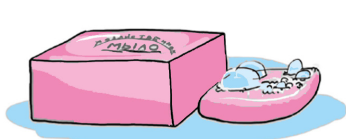

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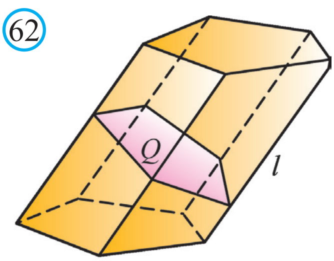
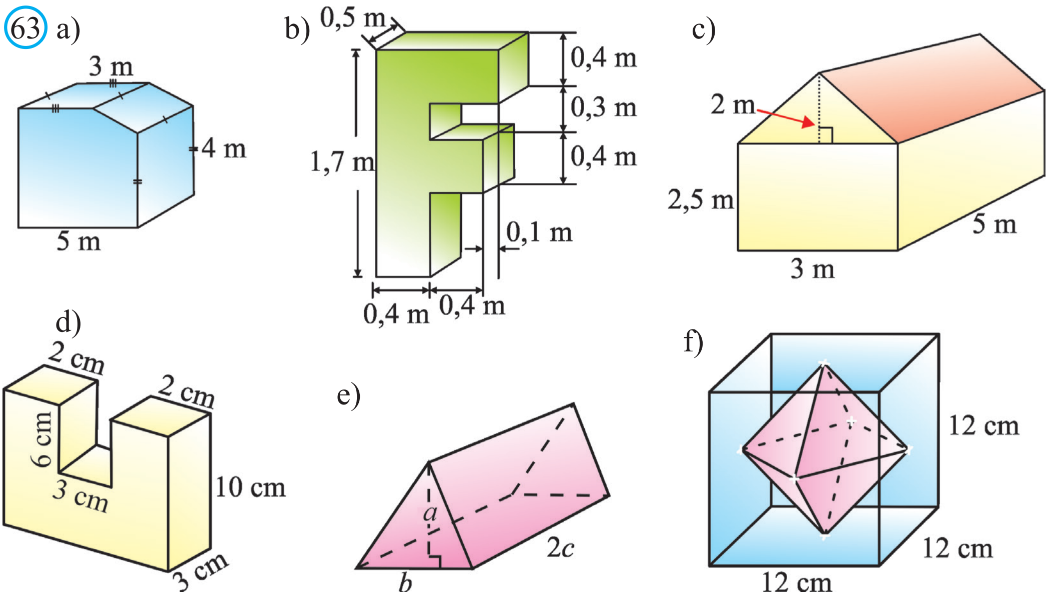


63 a)



245. Uchburchakli to‘g‘ri prizma asosining yuzi 4 cm^2 ga, yon yoqlarining yuzlari 9 cm^2, 10 cm^2, 17 cm^2 ga teng bo‘lsa, uning hajmini toping.

246*. Prizmaning asosi teng yonli uchburchak bo‘lib, uning bir tomoni 2 cm, qolgan ikki tomoni 3 cm ga teng. Prizmaning yon qirrasi 4 cm ga teng va u asos tekisligi bilan 45° li burchak tashkil etadi. Bu prizmaga tengdosh kubning qirrasini toping.

247. Og‘ma prizma asosining tomoni a ga teng bo‘lgan teng tomonli uchburchak. Yon yoqlaridan biri asosiga perpendikular va kichik diagonali s ga teng bo‘lgan rombdan iborat. Prizmaning hajmini toping.

248. Agar to‘rtburchakli to‘g‘ri prizmaning balandligi h, diagonallari asos tekisligi bilan α va β burchaklar tashkil qiladi. Agar asosining diagonallari orasidagi burchak γ ga teng bo‘lsa, prizmaning hajmini toping.

249*. Kesimi asosi 1,4 m va balandligi 1,2 m bo‘lgan teng yonli uchburchak shaklidagi suv chiqaruvchi quvurning suv o‘tkazish quvvatini (1 soatda oqib o‘tadigan suv hajmini) hisoblang. Suvning oqish tezligi 2 m/s.

250*. Temiryo‘l ko‘tarmasining kesimi trapetsiya shaklida bo‘lib, uning pastki asosi 14 m, yuqori asosi 8 m va balandligi 3,2 m. 1 km ko‘tarmani qurish uchun qancha kub metr tuproq kerak bo‘ladi?

251*. Tomoni 3,2 cm va qalinligi 0,7 cm bo‘lgan muntazam sakkizburchak shaklidagi yog‘och plitkaning massasi 17,3 g. Yog‘ochning zichligini toping.

252. O‘lchamlari 30×40×50 (cm) bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi qutidan nechtasini o‘lchamlari 2×3×1,5 m bo‘lgan mashina kuzoviga joylashishi mumkin?

253*. O‘lchamlari 420 mm × 240 mm × 90 mm bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi, zichligi 7,8 g/cm^3 bo‘lgan po‘lat plitalarning nechtasini yuk ko‘tarish quvvati 3 t bo‘lgan yuk mashinasida tashish mumkin?

254. O‘lchamlari 250 mm × 120 mm × 65 mm bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi, zichligi 1,6 g/cm^3 bo‘lgan g‘ishtning nechtasini yuk ko‘tarish quvvati 3 t bo‘lgan yuk mashinasiga yuklash mumkin?

255*. O‘lchamlari 820 mm × 210 mm × 120 mm bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi, zichligi 7,3 g/cm^3 bo‘lgan cho‘yan plitani yuk ko‘tarish quvvati 2 t bo‘lgan ko‘tarma kran yordamida ko‘tarish mumkinmi?

256. Bo‘yi 105 m va ko‘ndalang kesimi o‘lchamlari 30 cm × 40 cm bo‘lgan to‘g‘ri to‘rtburchakdan iborat yog‘ochdan, bo‘yi 3,5 m, eni 20 cm va qalinligi 20 mm bo‘lgan nechta taxta bo‘lagi chiqadi?

257. G‘ishtning o‘lchamlari 25 × 12 × 6,5 (cm). Agar 1 m^3 hajmdagi g‘ishtning massasi 1700 kg bo‘lsa, bir dona g‘ishtning massasini grammlarda aniqlang.

258. Sanitariya me’yorlariga ko‘ra, sinfdagi har bir o‘quvchiga 7,5 m^3 havo to‘g‘ri keladi. Agar sinfxonaning balandligi 3,5 m va u 28 o‘quvchiga mo‘ljallangan bo‘lsa, sinfxonaning maydonini toping.

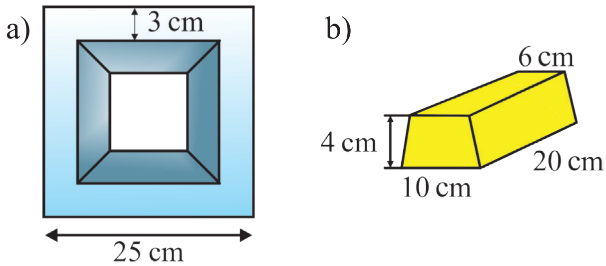
259*. Bo‘yi 100 m, eni esa 10 m bo‘lgan to‘g‘ri to‘rtburchak shaklidagi maydonni qalinligi 5 cm bo‘lgan asfalt bilan qoplash kerak. Agar 1 m^3 hajmdagi asfaltning massasi 2,4 tonna va bitta yuk mashinasining yuk ko‘tarish quvvati 5 tonna bo‘lsa, bu maydonni asfaltlash uchun nechta mashina asfalt kerak bo‘ladi?

260*. O‘lchamlari 3 cm, 4 cm, 5 cm bo‘lgan, to‘g‘ri burchakli paralle-lepiped shaklidagi temir parchasiga dastgohda ishlov berildi. Bu jarayonda uning har bir qirrasi birdek kamayib, to‘la sirti 42 cm^2 ga kamaygani ma’lum. Bu temir parchasining hajmi ishlov berilgandan keyin qanchani tashkil qiladi?

261*. 64.a- rasmda cho‘yan quvur kesimi tasvirlangan. Rasmda berilgan ma’lumotlar asosida bir metr uzunlikdagi bunday quvurning massasini aninqlang (cho‘yanning zichligi – 7,3 g/cm^3).

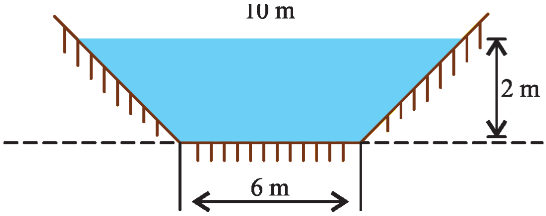
262. O‘lchamlari 64.b- rasmda berilgan oltin plitka (yombi) ning massasi 12,36 kg bo‘lsa, uning zichligini aniqlang.

64



263*. Kanalning ko‘ndalang kesimi asoslari 10 m, 6 m va balandligi 2 m bo‘lgan teng yonli trapetsiyadan iborat (65- rasm). Suv oqimi tezligi 1 m/s bo‘lsa, bir minutda bu kanaldan qancha hajmdagi suv oqib o‘tadi?

65



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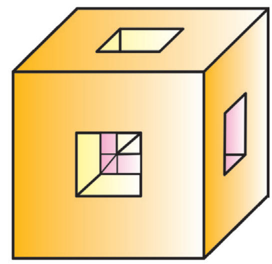
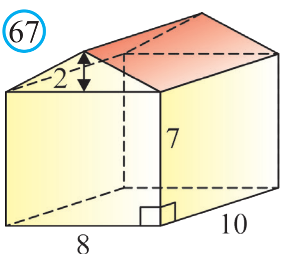


264*. Qirrasi 6 cm ga teng bo‘lgan, misdan ishlangan kubning har bir yog‘idan ko‘ndalang kesimi – asosi 2 cm ga teng kvadrat shaklidagi teshiklar o‘yilgan (66- rasm). Agar misning solishtirma zichligi 0,9 g/cm^3 bo‘lsa, kubning qolgan qismining massasini toping.

265. To‘g‘ri burchakli parallelepiped shaklidagi metall blok asosining o‘lchamlari 7 cm va 5 cm. Blokning massasi 1285 g va metalning zichligi 7,5 g/cm^3 bo‘lsa, blokning balandligini toping.

266. 67- rasmda berilgan ma’lumotlar asosida garajning hajmini toping.

67



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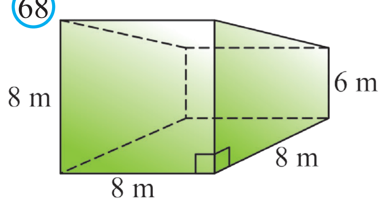

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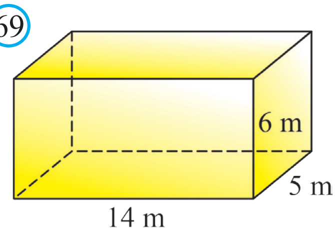


267. Gul o‘stiriladigan katta tuvak chuqurligi 2 fut, kengligi 12 fut va uzunligi 15 fut bo‘lgan to‘g‘ri burchakli parallelepiped shaklida. Tuvakning hajmini toping va kub metrlarda ifodalang (1 fut = 30,48 cm).

268. Yuk ombori 68- rasmda tasvirlangan trapetsiyali prizma shaklida. Rasmda berilgan ma’lumotlar asosida omborning sig‘imini aniqlang.

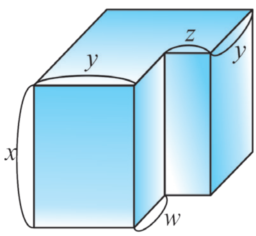
269*. 69- rasmda qutining o'lchamlari berilgan. Qutining asoslari 1 kvadrat metri 1000 so'm, yon yoqlari esa 1 kvadrat metri 2000 so'm bo'lgan materialdan ishlangan. Qutini yasashga necha so'mlik material ketgan?

270. Kubning hajmi V ga teng bo'lsa, uning diagonalini toping.

271. Katta to'g'ri burchakli parallelepipeddan 70- rasmda ko'rsatilgandek qilib kichik to'g'ri burchakli parallelepiped qirqib olingan. Berilgan ma'lumotlar asosida hosil bo'lgan jismning hajmini toping.

272. 71- rasmda tasvirlangan piramida hajmini toping.

70



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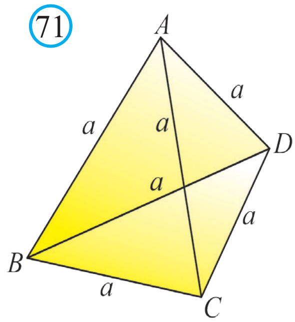

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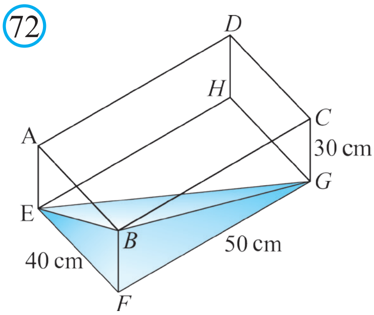
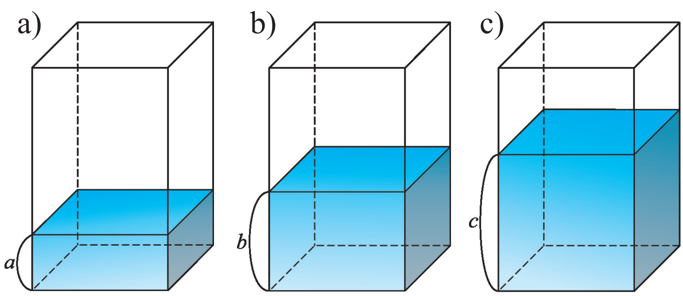


273*. 72- rasmda tasvirlangan to'g'ri burchakli parallelepiped shaklidagi akvariumda qancha suv bor?

274*. To'g'ri burchakli parallelipiped shaklidagi bir xil akvariumlarga 73- rasmda ko'rsatilgandek, turli sathdagi suv quyilgan. Bu akvariumlarga quyilgan suv hajmlarining nisbati qanday bo'ladi?

73



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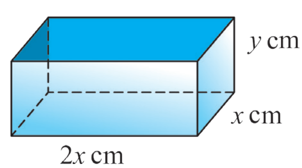


275*. Tatqiqot. Korxona sig'imi 1 litr, asosining o'lchamlari nisbati 1:2 bo'lgan to'g'ri burchakli parallelepiped shaklidagi usti ochiq qutilarni ishlab chiqarmoqchi (74- rasm). Qutini tejamli ishlab chiqarish, ya'ni unga ketadigan material eng kam bo'lishi uchun uning o'lchamlari qanday bo'lishi kerak? (x ga turli qiymatlar berib, qutining hajmini toping va ularni taqqoslash bilan yechishga urinib ko'ring yoki differensial hisob imkoniyatlaridan foydalaning.)

276*. **Muammoli vaziyat.** Geologlar tosh topib oldilar va uning hajmini taxminan bo‘lsada aniqlashmoqchi. Ular ko‘l yonida turishibdi va ularning ixtiyorida tosh sig‘adigan katta metall bak, bir nechta sig‘imi noma’lum chelaklar va sig‘imi 1 litr bo‘lgan butilka bor. Geologlar bu ishni qanday uddalay olishadi?

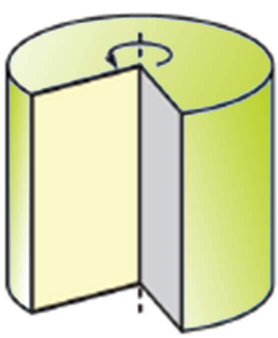
8. SILINDRNING SIRTI VA HAJMI

8.1. Silindrning sirti

Fazoviy shakllarning yana muhim sinflaridan biri – bu aylanish jismlaridir. Silindr aylanish jismlardan biri bo‘lib, u bilan quyi sinflarda tanishgansiz. Silindr xossalari prizmaning xossalariga o‘xshagani uchun ularni ketma-ket o‘rganamiz.

Tog‘ri to‘rtburchakni bir tomoni atrofida aylantirishdan hosil bo‘lgan jismga *silindr* (aniqrog‘i, to‘g‘ri doiraviy silindr) deb aytiladi (75- rasm). Bu aylanishda tog‘ri to‘rtburchakning bir tomoni qo‘zg‘alishsiz qoladi. Uni *silindrning o‘qi* deb ataymiz. To‘rtburchakning bu tomonga qarama-qarshi yotgan tomoni aylanishidan hosil bo‘lgan sirt – *silindrning yon sirti*, tomonning o‘zi esa *silindrning yasovchisi* deb ataladi. Tog‘ri to‘rtburchakni qolgan tomonlari bu aylanishda ikkita teng doira hosil qiladi, ularni *silindrning asoslari* deb ataymiz (76- rasm).

75



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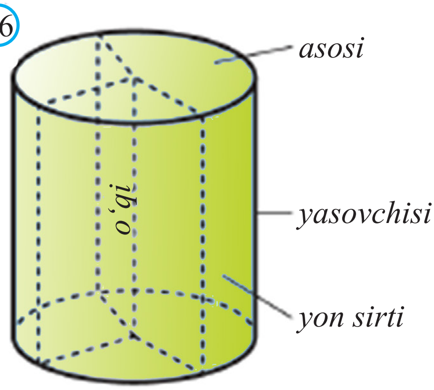


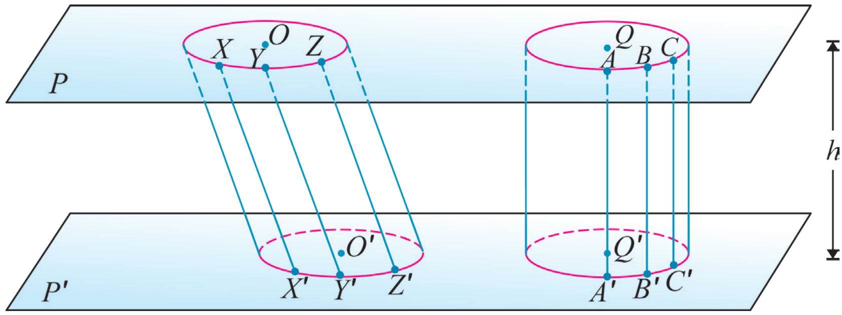
Eslatma. To‘g‘ri to‘rtburchakni bir tomoni atrofida aylantirishdan hosil bo‘lgan jism aslida *to‘g‘ri doiraviy silindr* deb yuritiladi. Silindr tushunchasi esa keng ma’noda quyidagicha kiritiladi.

Aytaylik, fazoda yassi F_1 shakl biror parallel ko‘chirishda F_2 shaklga o‘tsin. Bu ikki shakl va mazkur parallel ko‘chirishda bir-biriga o‘tgan nuqtalarni tutashtiruvchi kesmalardan iborat jismga *silindr* deb ataladi (77- rasm).

Agar parallel ko‘chirish yassi F_1 shakl tekisligiga perpendikular bo‘lsa, silindr *to‘g‘ri silindr* (78.a- rasm) deb, aks holda *og‘ma silindr* (78.b- rasm) deb yuritiladi.

78.c- rasmda tasvirlangan Piza minorasi og‘ma silindr shaklida.

77



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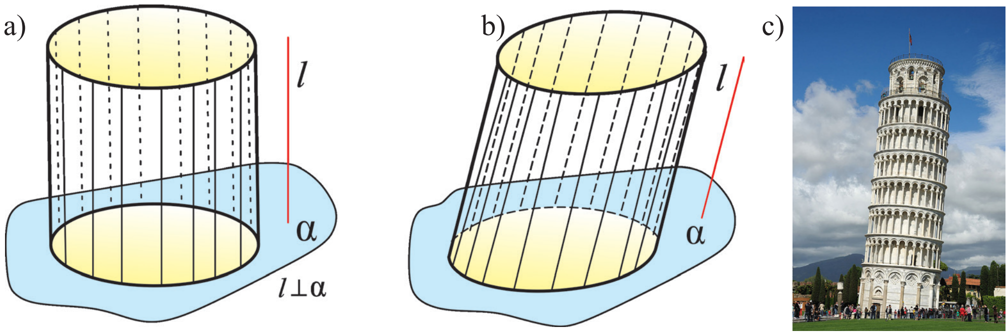


Agar F_1 shakl doiradan iborat bo‘lsa, silindr *doiraviy silindr* deb ataladi.

To‘g‘ri doiraviy silindrgina aylanma jism bo‘ladi. Kelgusida to‘g‘ri doiraviy silindrlar bilan ish ko‘ramiz va ularni qisqalik uchun silindrlar deb ataymiz.

Silindrning asoslari o‘zaro teng doiralardan iborat bo‘lib, ular parallel tekisliklarda yotadi. Silindrning bir asosi nuqtasidan ikkinchi asosi tekisligiga tushirilgan perpendikular uning *balandligi* deb ataladi.

Bu parallel tekisliklar orasidagi masofa silindrning balandligiga teng bo‘ladi. Silindrning o‘qi uning balandligi hamdir.

Silindrning yasovchilari esa o‘zaro parallel va teng bo‘ladi. Shuningdek, silindr o‘qi, yasovchilari va balandligi uzunliklari o‘zaro teng bo‘ladi.

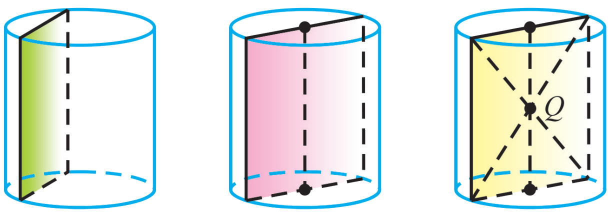
Silindrni uning o‘qiga parallel tekislik bilan kesganda hosil bo‘lgan kesim to‘g‘ri to‘rtburchakdan iborat bo‘ladi (79.a- rasm). Uning ikki tomoni silindrning yasovchilari, qolgan ikki tomoni esa mos ravishda asoslarning parallel vatarlaridir.

Xususan, *o'q kesim* ham to'g'ri to'rtburchak bo'ladi. U silindrning o'qi orqali o'tgan tekislik bilan kesganda hosil bo'lgan kesimdir (79.b- rasm).

O'q kesimlarning diagonallari asos markazlarini tutashtiruvchi kesmaning o'rtasi Q nuqtadan o'tadi. Shuning uchun, bu Q nuqta silindrning simmetriya markazidan iborat bo'ladi (79.c- rasm).

Q nuqtadan o'tuvchi va silindr o'qiga perpendikular bo'lgan tekislik silindrning simmetriya tekisligidan iborat bo'ladi (80- rasm). Silindrning o'qidan o'tuvchi tekisliklar ham uning simmetriya tekisliklari bo'ladi (81- rasm).

79 a) b) c)



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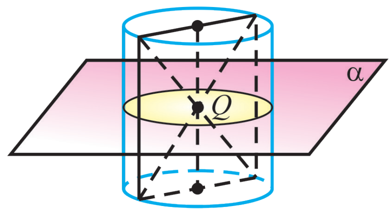


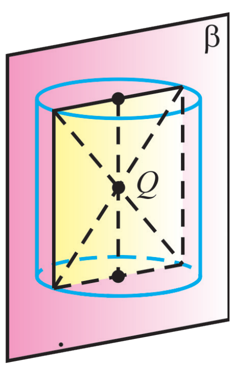
1- masala. Silindr o'q kesimining yuzi Q ga teng kvadratdan iborat. Silindr asosining yuzini toping.

Yechish. Kvadratning tomoni $\sqrt{Q}$ ga teng. U silindr asosining diametriga teng. Unda silindr asosining yuzi: $S = \pi r^2 = \pi\left(\frac{\sqrt{Q}}{2}\right)^2 = \frac{\pi Q}{4}$ ga teng. □

Teorema. Silindrning yon sirti asosining aylana uzunligi bilan yasovchisi ko'paytmasiga teng: $S_{yon} = 2\pi rl.$

Mazkur teoremani quyidagi 82- rasm asosida mustaqil isbotlang.

81



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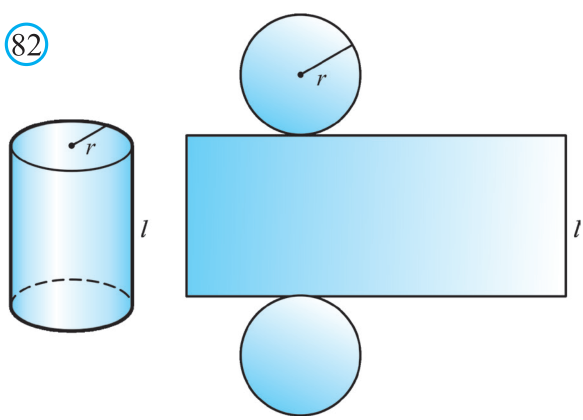


Natija. Silindr to‘la sirti uning yon sirti bilan ikkita asosining yuzi yig‘indisiga teng: $S_{to‘la} = S_{yon} + 2S_{asos}$ yoki

$$S_{to‘la} = 2\pi rl + 2\pi r^2 = 2\pi r\,(l + r).$$

Ixtiyoriy silindr berilgan bo‘lsin. Uning asoslaridan biriga ichki $A_1A_2...A_{n-1}A_n$ ko‘pburchakni chizamiz (83- rasm). Ko‘pburchakning $A_1, A_2, ..., A_{n-1}$ va A_n uchlari orqali, silindrning A_1B_1, A_2B_2, ..., $A_{n-1}B_{n-1}$ va A_nB_n yasovchilarni o‘tkazamiz hamda yasovchining boshqa $B_1, B_2, ..., B_{n-1}$ va B_n uchlarini ketma-ket kesmalar bilan tutashtirib chiqamiz. Natijada $A_1A_2...A_{n-1}A_n\,B_1B_2...B_{n-1}B_n$ prizmani hosil qilamiz. Bu prizma berilgan *silindrga ichki chizilgan prizma* deb ataladi. Silindr esa *prizmaga tashqi chizilgan silindr* deb yuritiladi. Agar prizma silindrga ichki chizilgan bo‘lsa, unda prizmaning asosi silindr asosiga ichki chizilgan bo‘ladi va prizmaning yon qirralari silindr yon sirtida yotadi.

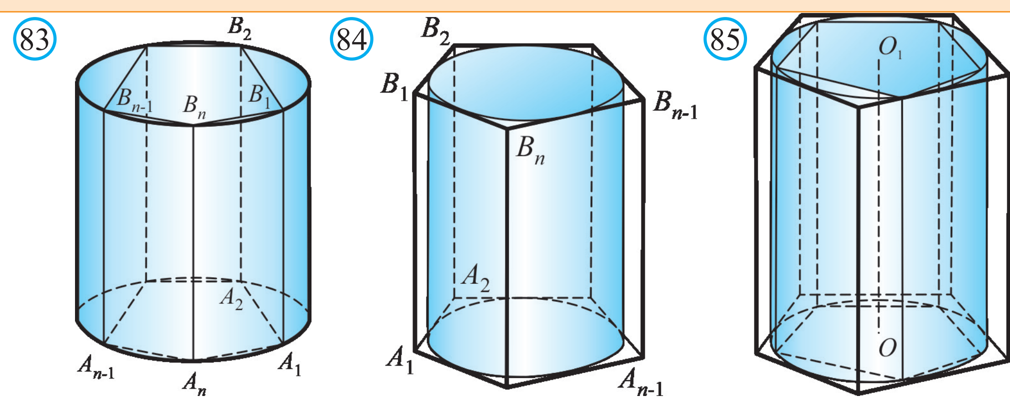
Ravshanki, agar prizma asosiga tashqi aylana chizish mumkin bo‘lsa, prizmaga tashqi silindr ham chizish mumkin.

Shunga o‘xshash *silindrga tashqi chizilgan prizma* va *prizmaga ichki chizilgan silindr* tushunchalari ham kiritiladi (84- rasm). Agar prizma silindrga tashqi chizilgan bo‘lsa, unda prizmaning asosi silindr asosiga tashqi chizilgan bo‘ladi va prizmaning yon yoqlari silindr yon sirtiga urinadi.

Ravshanki, agar prizma asosiga tashqi aylana chizish mumkin bo‘lsa, prizmaga tashqi silindr ham chizish mumkin.

8.2. Silindrning hajmi

Teorema. Silindrning hajmi asosining yuzi bilan yasovchisi ko‘paytmasiga teng: $V = S_{asos} \cdot l.$



Isbot. O‘qi OO_1 bo‘lgan silindr berilgan bo‘lsin (85- rasm).
Unga ichki $A_1A_2 ... A_{n-1}A_n\, B_1B_2 ... B_{n-1}B_n$ va tashqi $C_1C_2 ... C_{n-1}C_n$

$D_1 D_2 \dots D_{n-1} D_n$ prizmalarni chizamiz. Silindr hajmini V, ichki va tashqi chizilgan prizmalar hajmini V_1 va V_2 bilan belgilasak, unda $V_1 < V < V_2$ qo‘shtengsizlik o‘rinli bo‘ladi. Prizmalar hajmi quyidagi formulalardan topiladi:

$$V_1 = S_{A_1A_2 \dots A_{n-1}A_n} \cdot l \qquad \text{va} \qquad V_2 = S_{C_1C_2 \dots C_{n-1}C_n} \cdot l$$

Prizmalar asosi tomonlari soni n ni borgan sari oshirib boramiz. Unda ichki chizilgan prizma hajmi oshib boradi, tashqi chizilgan prizmaning hajmi esa kamayib boradi. Agar tomonlar soni n cheksiz kattalashib borsa, bu hajmlar orasidagi farq nolga intiladi. Silindrga ichki va tashqi chizilgan prizmalar hajmi yaqinlashgan son berilgan silindrning hajmi sifatida olinadi.

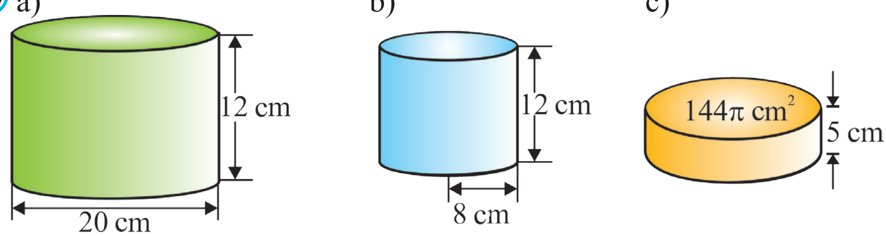
Bu jarayonda $A_1A_2 \dots A_{n-1}A_n$ va $C_1C_2 \dots C_{n-1}C_n$ ko‘pburchaklar yuzi silindr asosida yotgan doira yuzi S ga yaqinlashadi.

Demak, $V = S_{asos} \cdot l$. □

Mavzuga oid masalalar va amaliy topshiriqlar

277. 86- rasmda keltirilgan silindrlarning yon va to‘la sirtini toping.

86



278. Silindr asosining radiusi 6 cm, uning balandligi 4 cm. Silindr o‘q kesimining yuzini hisoblang.

279. Silindr asosining radiusi 2 m, balandligi 3 m. O‘q kesimining diagonalini toping.

280. Silindr asosining yuzi 64 π cm^2, uning balandligi 8 cm. Silindr o‘q kesimining yuzini hisoblang.

281. Silindrning o‘q kesimi – yuzi Q ga teng kvadrat. Silindr asosining yuzini toping.

282. Silindrning o‘q kesimi yuzi 36 cm^2 bo‘lgan kvadratdan iborat. Silindr yon sirtining yuzini hisoblang.

283. Silindr o‘q kesimining yuzi 4 ga teng. Uning yon sirti yuzini toping.

284. Silindrning balandligi 6 cm, asosining radiusi 5 cm. Silindrning o‘qiga parallel ravishda undan 4 cm masofada o‘tkazilgan kesimning yuzini toping.

285. Silindr asosining radiusi 2 ga, balandligi 3 ga teng. Silindr yon sirtining yuzini toping.

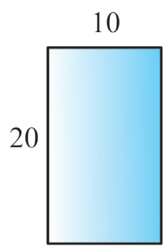
286. Silindr asosining aylana uzunligi 3π ga, balandligi 2 ga teng. Silindrning yon sirti yuzini toping.

287. Silindr yoyilmasining yuzi 24π dm^2, silindrning balandligi 4 dm. Uning asosi radiusini toping.

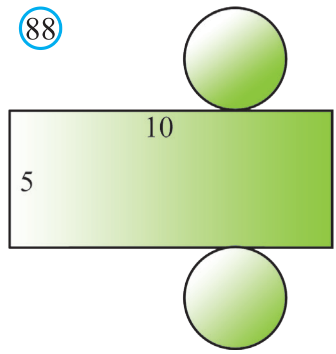
288. Silindr asosining radiusi 5 cm, uning balandligi 6 cm. Silindr o'q kesimining diagonalini toping.

289. Silindrning balandligi 8 dm, asosining radiusi 5 dm. Silindr tekislik bilan shunday kesilganki, kesimda kvadrat hosil bo'lgan. Bu kesimdan silindr o'qigacha bo'lgan masofani toping.

87



88



290*. 87- rasmda berilgan silindrning o'q kesimiga ko'ra, uning yon va to'la sirti yuzini toping.

291*. 88- rasmda berilgan silindrning yoyilmasiga ko'ra, uning yon va to'la sirtining yuzini toping.

292. Silindr asosining radiusi 3 cm, balandligi esa asos radiusidan 2 cm ortiq. Silindirning hajmini hisoblang.

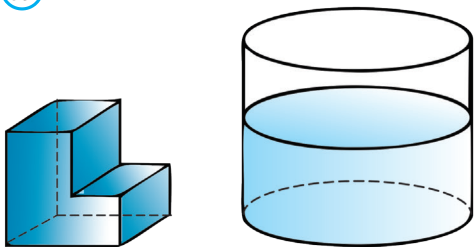
293. Silindrning hajmi 64 π cm^3, balandligi 4 cm. Silindr asosining yuzini hisoblang.

294*. Silindr shaklidagi idishga 2000 cm^3 suv solinganda suvning sathi 12 cm ni tashkil qildi. Idishga detal botirilganda esa suv sathi yana 9 cm ga ko'tarildi. Detal hajmini aniqlang va javobni cm^3 larda ifodalang.

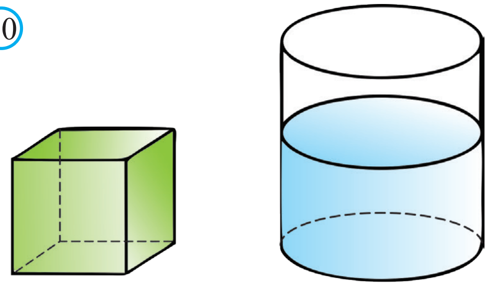
295. Silindr shaklidagi idishga 3 litr suv solinganda suvning sathi 15 cm ni tashkil qildi (89-rasm). Idishga detal botirilganda esa suv sathi yana 4 cm ga ko'tarildi. Detal hajmini aniqlang va javobni cm^3 larda ifodalang.

296*. Silindr shaklidagi idishga 4 litr suv solinganda suvning sathi 20 cm ni tashkil qildi (90-rasm). Idishga detal botirilganda esa suv sathi yana 5 cm ga ko'tarildi. Detal hajmini aniqlang va javobni cm^3 larda ifodalang.

89



90

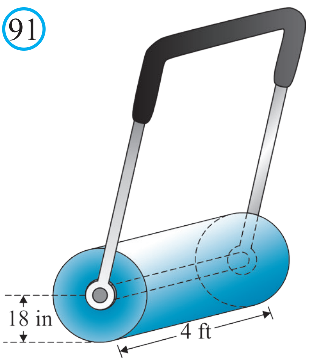


297*.91- rasmda silindr shaklidagi yo‘l tekislagich moslamasi tasvirlangan. Rasmda berilganlardan foydalanib, u bir marta aylanganda qancha maydondagi yo‘lni tekislashini aniqlang.

(Eslatma: 1 ft (fut) = 12 in. (dyuym) = 30,48 cm).

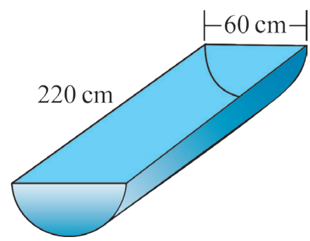
298*.92- rasmdagi suv sepishga mo‘ljallangan rezina quvurning ichki diametri 3 cm, tashqi diamerti 3,5 cm, uzunligi esa 20 m bo‘lsa, unga necha lirt suv ketishini toping. Agar rezinaning zichligi 7 g/cm^3 ekanligi ma’lum bo‘lsa, bu rezina quvur o‘ramining massasini toping.

91



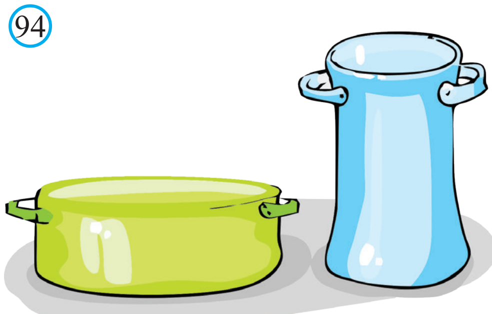
92

93



299*.93- rasmda yon sirti yarim silindr shaklida bo‘lgan idish berilgan. Agar 1 cm^2 yuzali sirtni bo‘yash uchun 6 g bo‘yoq talab etilsa, bu idishning ham ichki, ham tashqi qismini bo‘yash uchun qancha bo‘yoq kerak bo‘ladi? Idishga necha litr suv ketadi?

94



95

96

300*. Silindr shaklidagi idishlardan biri ikkinchisidan ikki marta kengroq, lekin uch marta pastroq (94- rasm). Bu idishlarning qaysi birining sig‘imi katta?

301*. Asosining radiusi 5 cm, balandligi esa 20 cm bo‘lgan silindr shaklidagi apelsin sharbati idishining asoslari metaldan, yon sirti esa kartondan ishlangan (95- rasm). Agar 1 cm^2 metall narxi 5 so‘m, 1 cm^2 karton narxi esa 2 so‘m bo‘lsa, bu idishni tayyorlash uchun necha so‘mlik material kerak bo‘ladi? Idishga qancha apelsin sharbati ketadi?

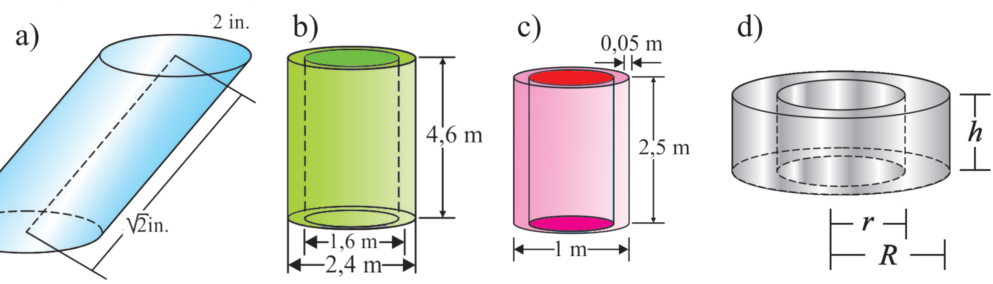
302*. Asosining radiusi 1,5 dyuym, balandligi esa 4,25 dyuym bo‘lgan silindr shaklidagi konserva bankasi berilgan (96- rasm). Bankaning to‘la sirti va hajmini toping. Agar 1 cm^2 metall narxi 5 so‘m bo‘lsa, bu idishni tayyorlash uchun necha so‘mlik material kerak bo‘ladi? (Eslatma: 1 in. (dyuym) = 2,54 cm.)

303*. Neft saqlanadigan idish (sisterna) balandligi 16 fut, asosining radiusi 10 fut bo‘lgan silindr shaklida. Agar 1 kub fut 7,5 gallonga teng bo‘lsa, bu sisternaning gallonlardagi sig‘imini aniqlang. (Eslatma: 1 amerika galloni = 3,785 litr. 1 amerika barelli = 42 amerika galloni = 159 litr.)

304*. Fermerning yoqilg‘i baki silindr shaklida. Bakning balandligi 6 fut, asosining radiusi 1,5 fut. Bakning gallonlardagi sig‘imini aniqlang.

305. 97- rasmdagi ma’lumotlardan foydalanib, tasvirlangan fazoviy jismlar hajmini aniqlang.

97

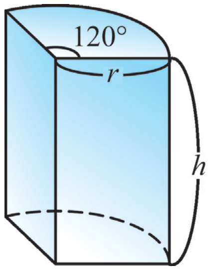


306*. Silindr shaklidagi idishga 6 cm^3 suv solindi. Idishga detal to‘liq cho‘ktirilganda, suv sathi 1,5 marta ko‘tariladi. Detall hajmini aniqlang va javobni cm^3 larda ifodalang.

307*. Silindr shaklidagi idishdagi suvning sathi 16 cm. Idishga asosining diametri bu idishga qaraganda 2 marta kichik bo‘lgan silindr shaklidagi ikkinchi idish botirilganda undagi suvning sathi qancha bo‘ladi?

308. Birinchi silindr hajmi 12 m^3. Ikkinchi silindrning balandligi birinchi

98



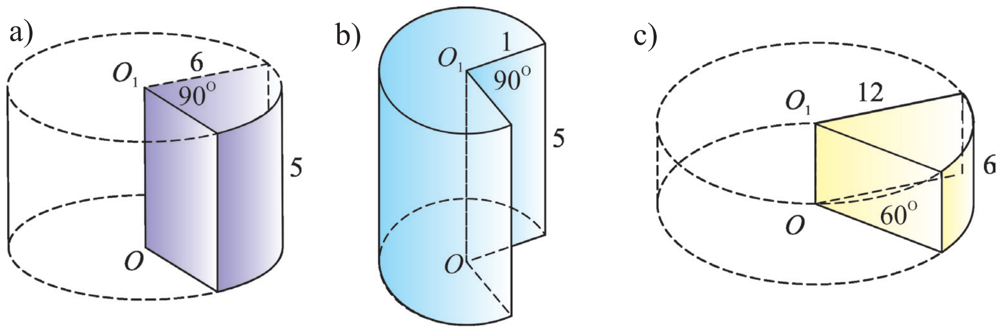
silindrga qaraganda 3 marta katta, asosining radiusi esa 2 marta kichik. Ikkinchi silindr hajmini toping.

309*. Silindr shaklidagi idish ikkinchisidan 2 marta baland, lekin 1,5 marta kengroq. Bu idishlar hajmlarining nisbatini hisoblang.

310. 98- rasmda tasvirlangan fazoviy jism hajmini toping.

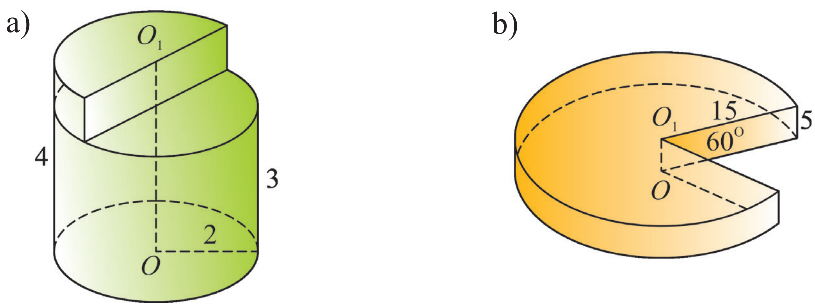
311. 99- rasmda tasvirlangan silindr bo'lagining hajmini toping.

99



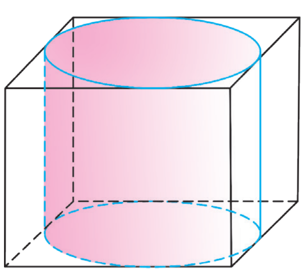
312. 100- rasmda tasvirlangan silindr bo'lagining hajmini toping.

100

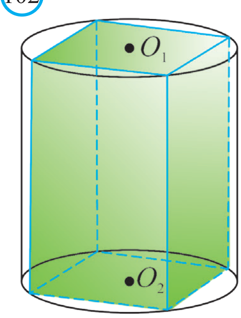


313. To'g'ri burchakli parallelepiped asosining radiusi va balandligi 1 ga teng bo'lgan silindrga tashqi chizilgan (101- rasm). Parallelepiped hajmini toping.

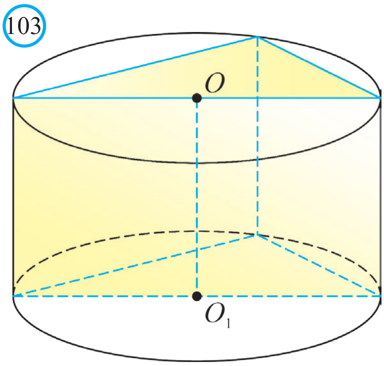
101



102



103



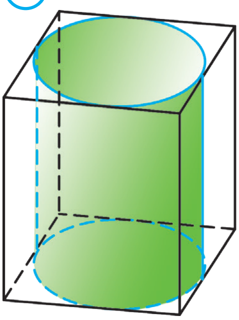
314. To‘g‘ri burchakli parallelepiped asosining radiusi 4 ga teng bo‘lgan silindrga tashqi chizilgan (102- rasm). Parallelepiped hajmi 16 ga teng bo‘lsa, silindrning balandligini toping.

315. To‘g‘ri prizmaning asosi katetlari 6 va 8 bo‘lgan to‘g‘ri burchakli uchburchakdan iborat, yon qirralari esa 5 ga teng (103- rasm). Bu prizmaga tashqi chizilgan silindr hajmini toping.

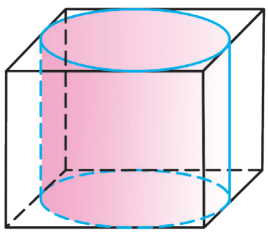
316. To‘g‘ri prizmaning asosi – tomoni 2 ga teng bo‘lgan kvadratdan iborat, yon qirralari esa 2 ga teng. Bu prizmaga tashqi chizilgan silindr hajmini toping.

317. To‘rtburchakli to‘g‘ri prizma asosining radiusi 2 ga teng bo‘lgan silindrga tashqi chizilgan (104- rasm). Prizma yon sirtining yuzi 48 ga teng bo‘lsa, silindrning balandligini toping.

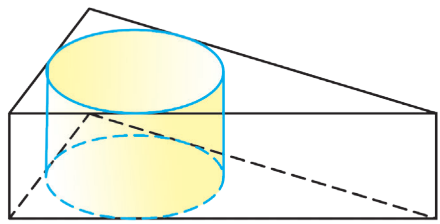
104



105



106

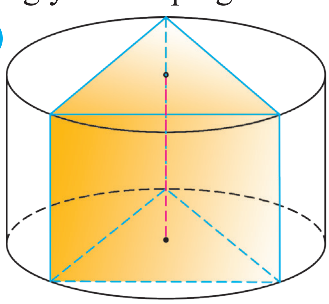


318. Muntazam to‘rtburchakli prizma asosining radiusi va balandligi 1 ga teng bo‘lgan silindrga tashqi chizilgan (105- rasm). Prizma yon sirtining yuzini toping.

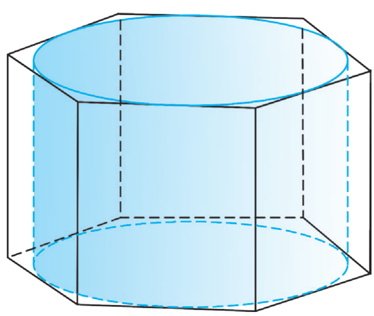
319. Uchburchakli to‘g‘ri prizma asosining radiusi $\sqrt{3}$ ga va balandligi 2 ga teng bo‘lgan silindrga tashqi chizilgan (106- rasm). Prizma yon sirtining yuzini toping.

320. Uchburchakli muntazam prizma asosining radiusi $2\sqrt{3}$ ga va balandligi 2 ga teng bo‘lgan silindrga ichki chizilgan (107- rasm). Prizma yon sirtining yuzini toping.

107



108



321. Oltiburchakli muntazam prizma asosining radiusi $\sqrt{3}$ ga va balandligi 2 ga teng bo‘lgan silindrga tashqi chizilgan (108- rasm). Prizma yon sirtining yuzini toping.

322*. 109- rasmda tasvirlangan detalning hajmini toping.

323*. Uzunligi 10 m, asosining diametri 1 m bo‘lgan silindr shaklidagi quvurning tashqi sirtini 1 mm qalinlikdagi bo‘yoq bilan bo‘yash uchun qancha bo‘yoq kerak bo‘ladi?

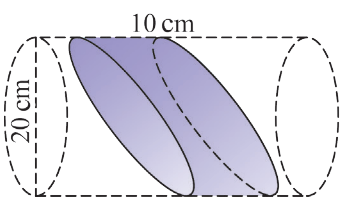
324*. 110- rasmda tasvirlangan tirsakli quvurning: a) yon sirtining yuzini; b) hajmini toping ($\pi \approx 3$ deb oling).

325*. Cho‘yan quvurning uzunligi 2 m, tashqi diametri 20 cm. Quvur devorining qalinligi 2 cm va cho‘yannning solishtirma zichligi 7,5 g/cm^3 bo‘lsa, uning massasini toping.

326*. 111- rasmdan foydalanib, og‘ma silindr uchun $S \cdot h = Q \cdot l$ tenglik o‘rinli bo‘lishini asoslang.

327*. 112- rasmda tasvirlangan silindr sirtidan A nuqtadan B nuqtaga olib boradigan eng qisqa yo‘lning uzunligini toping. (Ko‘rsatma: silindr yoyilmasidan foydalaning.)

109



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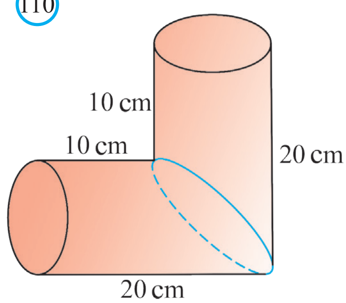

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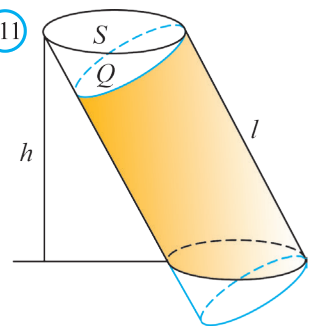

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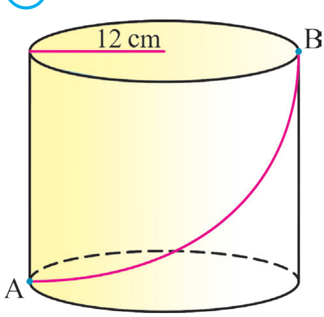


Tarixiy ma'lumotlar

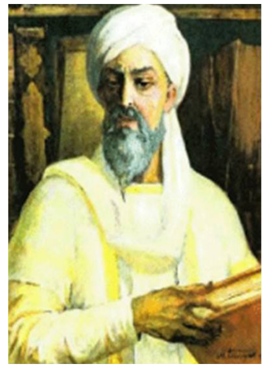
Abu Rayhon Beruniyning "Astronomiya san'atidan boshlang'ich ma'lumot beruvchi kitob" (qisqacha "Astronomiya") nomli asarining geometriyaga tegishli qismida stereometriyaga kirish sifatida fazoviy shakllarning quyidagi ta'riflari keltiriladi.

Kub – jismiy shakl bo'lib, nardning soqqasiga o'xshaydi, oltita tomonidan oltita kvadrat bilan chegaralangan.

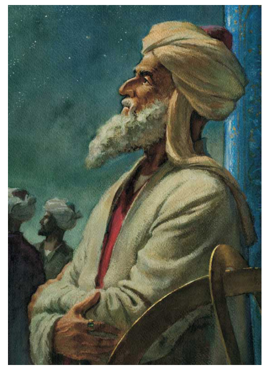
Prizma – mujassam shakl bo'lib, yon tomonidan kvadrat yoki to'g'ri to'rtburchak shaklidagi tekisliklar bilan, osti va ustidan ikkita uchburchak bilan chegaralangan.

Beruniy bergan bu ta'rifda prizmaning xususiy holi, ya'ni uchburchakli prizmaning ta'rifi keltirilgan.

Abu Rayhon Beruniyning "Qonuni Ma'sudiy" kitobi 1037- yilda yozilgan bo'lib, unda parallelepiped, prizmaning hajmlarini topish qoidalari: "Agar jism to'rtburchakli bo'lmasdan yoki boshqa xil bo'lsa, uning o'lchami quyidagicha: uning yuzini bilgin, uni chuqurlikka ko'paytirgin, natijada hajm hosil bo'ladi" tarzda berilgan.



Abu Ali ibn Sino



G'iyosiddin al Koshiy

Abu Ali ibn Sino "Donishnoma" nomli asarining "Geometrik jismlarga oid negizlar" bobida jismning va uchburchakli prizmaning ta'rifini beradi hamda ikki prizmaning o'zaro teng bo'lish shartlarini bayon qiladi. Ibn Sino prizmani quyidagicha ta'riflaydi: "Prizma ikkita uchburchakli tekis shakllar va tomonlari o'zaro parallel uchta tekis shakllar bilan chegaralangan jism-dir".

G'iyosiddin Jamshid ibn Ma'sud al- Koshiyning "Hisob kitobi" nomli asarida sirtlar yuzlarini va jismlarning hajmlarini hisoblashning ko'plab qoidalari keltirilgan. U matematika, geometriya, trigonometriya, mexanika va astronomiya kabi fanlarni chuqur bilganligi uchun Ulug'bekning e'tibori va hurmatiga sazovor bo'lgan. Al- Koshiy ko'pburchaklar bilan bir qatorda prizmalar, piramidalar, silindrlar, konuslar, kesik konuslarni ham tadqiq qilgan.

9. BOBNI TAKRORLASHGA DOIR AMALIY MASHQLAR

9.1. 2- test sinovi

1. Kubning nechta simmetriya tekisligi mavjud?
A) 8; B) 9; C) 7; D) 10.

2. Agar kub diagonal kesimining yuzi $2\sqrt{2}$ ga teng bo‘lsa, uning hajmini toping.
A) $2\sqrt{2}$; B) $\sqrt{7}$; C) $4\sqrt{2}$; D) $5\sqrt{2}$.

3. To‘g‘ri burchakli parallelepiped asosining tomonlari 7 cm va 24 cm. Parallelepipedning balandligi 8 cm. Diagonal kesimining yuzini toping.
A) 168; B) 1344; C) 100; D) 200.

4. Muntazam to‘rtburchakli prizmaning diagonali 4 ga teng bo‘lib, yon yog‘i bilan 300 li burchak tashkil qiladi. Prizmaning yon sirtini toping.
A) $16\sqrt{2}$; B) 16; C) 18; D)$18\sqrt{2}$.

5. Muntazam to‘rtburchakli prizma asosining tomoni $\sqrt{2}$ ga, diagonali bilan yon yog‘i orasidagi burchak esa 300 ga teng. Prizmaning hajmini toping.
A) $8\sqrt{2}$; B) 4; C) 16; D) $4\sqrt{2}$.

6. Prizmaning jami qirralari 36 ta bo‘lsa, uning nechta yon yog‘i bor?
A) 12; B) 16; C) 9; D) 10.

7. Og‘ma prizmaning yon qirrasi 20 ga teng va asos tekisligi bilan 300° li burchak hosil qiladi. Prizmaning balandligini toping.
A) 12; B) $10\sqrt{3}$; C) 10; D) $10\sqrt{2}$.

8. Uchburchakli to‘g‘ri prizma asosining tomonlari 15, 20 va 25 ga, yon qirrasi asosining balandligiga teng. Prizmaning hajmini toping.
A) 600; B) 750; C) 1800; D) 1200.

9. Muntazam oltiburchakli prizmaning eng katta diagonali 8 ga teng va u yon qirrasi bilan 300° li burchak hosil qiladi. Prizmaning hajmini toping.
A) 72; B) 64; C) 76; D) 80.

10. O‘q kesimining yuzi 10 ga teng bo‘lgan silindr yon sirtining yuzini toping.
A) 10π; B) 20π; C) 30π; D) 15π.

11. Silindrning balandligi 8 ga yon sirti yoyilmasining diagonali 10 ga teng. Silindr yon sirtining yuzini toping.
A) 48; B) 48π; C) 24; D) 48π.

12. Tomonlari 2 va 4 ga teng bo‘lgan to‘g‘ri to‘rtburchak o‘zining katta tomoni atrofida aylandi. Hosil bo‘lgan jismning to‘la sirtini toping.
A) 22π; B) 23π; C) 24π; D) 20π.

13. Silindrning yon sirti yuzi $72\ \pi$ ga teng va u yoyilganda hosil bo‘lgan to‘g‘ri to‘rtburchak diagonali asosi bilan 45° burchak tashkil qiladi. Silindr asosining radiusini toping.

A) 5; B) 4; C) 6; D) 8.

14. Silindr asosining radiusi ikki marta orttirilsa, uning hajmi necha marta ortadi?

A) 4; B) 2; C) 3; D) 6.

15. Silindrning hajmi 120π ga, yon sirti 60π ga teng. Silindr asosining radiusini toping.

A) 4; B) 5; C) 6; D) 4; 2.

16. Silindrning balandligi 5 ga, asosiga ichki chizilgan muntazam uchburchakning tomoni $3\sqrt{3}$ ga teng. Silindrning hajmini toping.

A) 25π; B) 35π; C) 45π; D) 40π.

17. Silindrning o‘q kesimi diagonali 12 ga teng bo‘lgan kvadratdan iborat. Uning hajmini toping.

A) $108\sqrt{2}\pi$; B) $54\sqrt{2}\pi$; C) $36\sqrt{2}\pi$; D) $216\sqrt{2}\pi$.

18. Silindrning to‘la sirti 24π ga, yon sirti esa 6π ga teng. Shu silindrning hajmini toping.

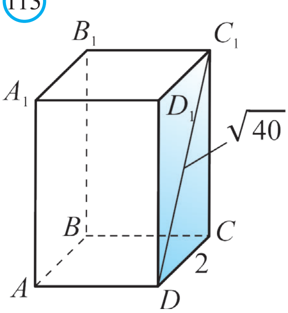
A) 7π; B) 11π; C) 8π; D) 9π.

9.2. Masalalar

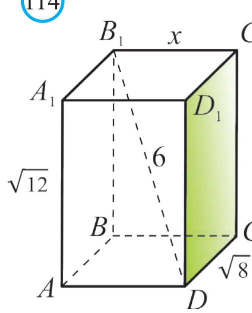
328. $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepipedda (113- rasm) $DC_1=\sqrt{40}$, $DC=2$, $P_{ABCD}=10$. Parallelepipedning diagonalini toping.

329. $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepiped. 114- rasmda berilgan ma’lumotlarga ko‘ra B_1C_1 qirraning uzunligini toping.

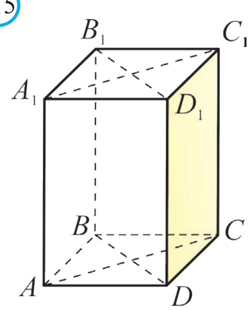
113



114

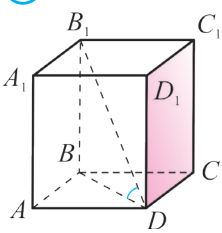


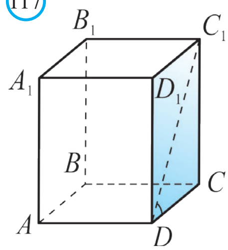
115

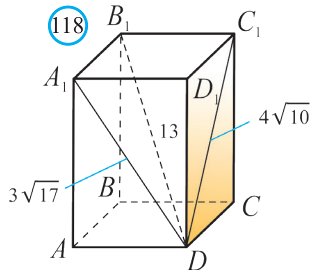


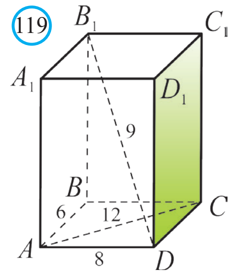
330. To‘g‘ri prizmaning asosi *ABCD* romb (115- rasm). Prizmaning diagonal kesimlari yuzi 60 va 80 ga, balandligi esa 10 ga teng. Prizmaning yon sirtini toping.

331. To‘g‘ri prizmaning asosi *ABCD* romb. Prizmaning diagonal kesimlari yuzi 24 va 32 ga, balandligi esa 4 ga teng. Prizmaning yon sirtini toping.

116


117


118


119


332. $ABCDA_1B_1C_1D_1$ muntazam prizma (116-rasm)da $\angle B_1DB = 45°$, $S_{\text{to'la}} = 32(2\sqrt{2}+1)$. AD ni toping.

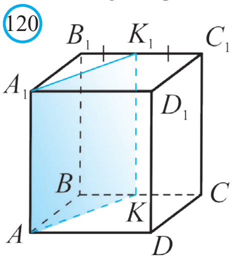
333. $ABCDA_1B_1C_1D_1$ muntazam prizma (117-rasm)da $\angle C_1DC = 60°$, $S_{\text{to'la}} = 128(2\sqrt{3}+1)$. AD ni toping.

334. $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepiped (118- rasm)da $DB_1 = 13$, $DA_1 = 3\sqrt{17}$, $DC_1 = 4\sqrt{10}$. Parallelepiped yon sirtining yuzini toping.

335. $ABCDA_1B_1C_1D_1$ to‘g‘ri parallelepiped (119- rasm)da $AB = 6$, $AD = 8$, $DB_1 = 9$. Parallelepiped yon sirtining yuzini toping.

336. K nuqta BC qirraning o‘rtasi (120- rasm). $ABKA_1B_1K_1$ prizma hajmining $ABCDA_1B_1C_1D_1$ parallelepiped hajmiga nisbatini toping.

337. N va M nuqtalar parallelepiped qirralarining o‘rtalari (121- rasm). $AA_1B_1NDD_1C_1M$ prizma hajmining $ABCDA_1B_1C_1D_1$ parallelepiped hajmiga nisbatini toping.

120


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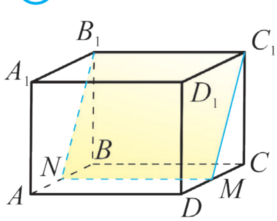

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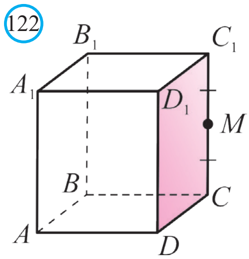
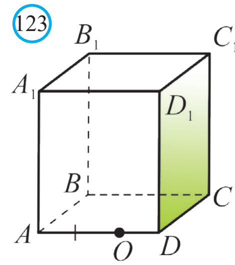


123


338. To‘rtburchakli muntazam prizma yon sirtining yuzi 72 cm^2 ga, asosining yuzi esa 64 cm^2 ga teng. Prizmaning hajmini toping.

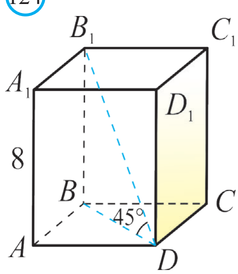
339. To‘rtburchakli muntazam prizma asosining perimetri 12 cm, yon yog‘ining perimetri esa 18 cm ga teng. Prizmaning hajmini toping.

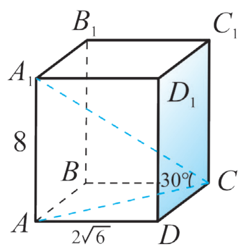
340. Kub berilgan (122- rasm). $CM = MC_1$ va ADM tekislik kubni ikki bo‘lakka ajratadi. Kubning katta bo‘lagi hajmining kichik bo‘lagi hajmiga nisbatini toping.

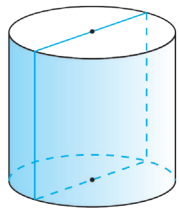
341*. Kub berilgan (123- rasm). $AO : OD = 2 : 1$ va BB_1O tekislik kubni ikki bo‘lakka ajratadi. Agar kubning kichik bo‘lagi hajmi 6 ga teng bo‘lsa, kubning hajmini toping.

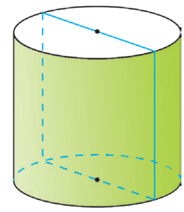
342*. To‘rtburchakli muntazam prizmaning balandligi 8 ga, diagonalining asos tekisligiga qiyaligi 45° ga teng (124- rasm). Prizmaning hajmini toping.

343*. To‘rtburchakli muntazam prizmada asosining tomoni $2\sqrt{6}$ ga, diagonali asos tekisligi bilan 30° li burchak tashkil qiladi (125- rasm). Prizmaning hajmini toping.

124


125


126


127


344. Silindr yon sirtining yuzi 91π ga teng (126- rasm). Silindr o‘q kesimining yizini toping.

345. Silindr o‘q kesimi yuzi 173 ga teng bo‘lgan kvadrat (127- rasm). Silindr yon sirtining yuzini toping.

346. Silindr balandligi 24 ga, o‘q kesim diagonali 26 ga teng. Silindr hajmini toping.

347. Silindr o‘q kesimi yuzi 10 ga. Asos aylanasining uzunligi 8 ga teng. Silindr hajmini toping.

348. Silindr radiusi 3 ga, yon sirtining yuzi 200 ga teng. Silindr hajmini toping.

9.3. 2-nazorat ishi namunasi

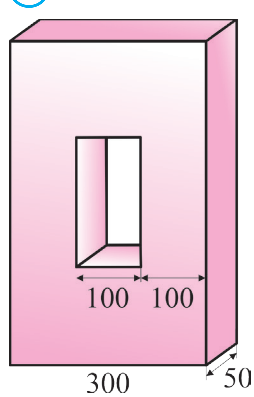
1. Ikkiyoqli burchakning *A* nuqtasi uning qirrasidan 10 cm, yog‘idan 5 cm uzoqlikda joylashgan. Ikkiyoqli burchakning gradus o‘lchovini toping.

2. Oltiburchakli muntazam prizmaning barcha qirralari 2 ga teng bo‘lsa, uning to‘la sirtining yuzini toping.

3. Asosining diamerti 18 m va balandligi 7 m bo‘lgan silindr shaklidagi sisterna neft bilan to‘ldirilgan. Agar neftning zichligi 0,85 g/cm^3 bo‘lsa, bu sisternadagi neftning massasi necha tonna?

4. Har bir qirrasi uzunligi 4 cm ga teng bo‘lgan muntazam oltiburchakli prizmaga ichki chizilgan silindr hajmini toping.

5. (*Yaxshi o‘zlashtiradigan o‘quvchilar uchun qo‘shimcha masala.*) 128- rasmda o‘lchamlar mm larda berilgan detalning to‘la sirti va hajmini toping.

128


Trigonometrik funksiyalarning taqribiy qiymatlari jadvali

A	sin *A*	tg *A*	*A*	sin *A*	tg *A*	*A*	sin *A*	tg *A*
0°	0	0	30°	0,50	0,58	60°	0,87	1,73
1°	0,0175	0,0175	31°	0,52	0,60	61°	0,87	1,80
2°	0,035	0,035	32°	0,53	0,62	62°	0,88	1,88
3°	0,05	0,05	33°	0,54	0,65	63°	0,89	1,96
4°	0,07	0,07	34°	0,56	0,68	64°	0,90	2,02
5°	0,09	0,09	35°	0,57	0,70	65°	0,91	2,15
6°	0,10	0,11	36°	0,59	0,73	66°	0,91	2,25
7°	0,12	0,12	37°	0,60	0,75	67°	0,92	2,36
8°	0,14	0,14	38°	0,62	0,78	68°	0,93	2,48
9°	0,16	0,16	39°	0,63	0,81	69°	0,93	2,61
10°	0,17	0,18	40°	0,64	0,84	70°	0,94	2,78
11°	0,19	0,19	41°	0,66	0,87	71°	0,95	2,90
12°	0,21	0,21	42°	0,67	0,9	72°	0,95	3,08
13°	0,23	0,23	43°	0,68	0,93	73°	0,96	3,27
14°	0,24	0,25	44°	0,69	0,97	74°	0,96	3,49
15°	0,26	0,27	45°	0,71	1,00	75°	0,97	3,73
16°	0,28	0,29	46°	0,72	1,04	76°	0,97	4,01
17°	0,29	0,31	47°	0,73	1,07	77°	0,97	4,33
18°	0,31	0,32	48°	0,74	1,11	78°	0,98	4,71
19°	0,33	0,34	49°	0,75	1,15	79°	0,98	5,15
20°	0,34	0,36	50°	0,77	1,19	80°	0,98	5,67
21°	0,36	0,38	51°	0,78	1,23	81°	0,99	6,31
22°	0,37	0,40	52°	0,79	1,28	82°	0,99	7,12
23°	0,39	0,42	53°	0,80	1,33	83°	0,992	8,14
24°	0,41	0,45	54°	0,81	1,38	84°	0,994	9,51
25°	0,42	0,47	55°	0,82	1,43	85°	0,996	11,43
26°	0,44	0,49	56°	0,83	1,48	86°	0,998	14,30
27°	0,45	0,51	57°	0,84	1,54	87°	0,999	19,08
28°	0,47	0,53	58°	0,85	1,60	88°	1,00	28,64
29°	0,48	0,55	59°	0,86	1,66	89°	1,00	57,29

JAVOBLAR

1- bob javoblari

3. $A(5; 7; 10)$, $B(4; -3; 6)$, $C(5; 0; 0)$, $D(4; 0; 4)$, $E(0; 5; 0)$, $F(0; 0; -2)$. **6.** $(3; 2; 0)$, $(3; 0; 4)$, $(0; 2; 4)$. **8.** $\sqrt{26}$. **9.** a) 3, 3, 3; b) $3\sqrt{2}, 3\sqrt{2}, 3\sqrt{2}$; c) $3\sqrt{2}$. **10.** 2, 3, 1. **11.** $(3; 3; 3)$, $(-3; 3; 3)$, $(3; -3; 3)$, $(3; 3; -3)$, $(-3; -3; 3)$, $(-3; 3; -3)$, $(3; -3; -3)$, $(-3; -3; -3)$. **12.** $O(0; 0; 0)$, $B(2; 0; 0)$, $A(2; 2; 0)$, $C(0; 2; 0)$, $O_1(0; 0; -2)$, $B_1(2; 0; -2)$, $A_1(2; 2; -2)$, $C_1(0; 2; -2)$. **13.** D nuqta. **14.** $3\sqrt{6}$. **15.** Yo‘q. **17.** *c)* teng yonli, $P=6\ (1+\sqrt{3})$, $S = 9\sqrt{2}$. **18.** $(-0,25; 0,25; 0)$. **19.** $D_1(1; -1; 1)$, $A_1(1; 1; -1)$, $B_1(-1; 1; -1)$, $D_1(1; -1; -1)$. **21.** $x^2+y^2+z^2=25$, $x^2+y^2+z^2\le 25$. **22.** $(x-1)^2+(y-2)^2+(z-4)^2=9$; $(x-1)^2+(y-2)^2+(z-4)^2\le 9$. **23.** $(x+2)^2+(y-3)^2+(z-4)^2=9$. **25.** 1)$(0; 1; 0)$; 2) $(1; 1; 1)$; 3) $(0; 0; 2)$, 4) $(-0,7; 0,1; 0,6)$; 5) $(2\sqrt{3}; 1,5; 1)$. **28.** $A(5;-4;0)$, $B(-7;5;6)$, **31**. $K\left(0;-5;\dfrac{17}{2}\right)$. **32.** a) $D(-1; -3; -9)$.

33. *a)* $M(-1; 2; 0)$; *c)* $M(3;\frac{3}{4}; 0)$. **35.** $L(\frac{25}{8};\frac{33}{8};\frac{9}{4})$. **36.** $\frac{4\sqrt{2}}{5}$. **37**. a) $\sqrt{2}$; b) 30°; 30°; 120°; c) $2\sqrt{3}$. **38.** $MK=\frac{\sqrt{73}}{3}$. **39.** $A(5; 4; 10)$, $B(4; -3; 6)$, $C(5; 0; 0)$, $D(4; 0; 4)$. **40.** $\overline{OA}=(1; 1; 1)$, $\overline{OB}=(-1; 0; 1)$, $\overline{OC}=(0; 1; 1)$, $\overline{BO}=(1; 0; -1)$, $\overline{CO}=(0; -1; -1)$, $\overline{AB}=(-2; -1; 0)$. **42.** a) $\overline{AB}=(2; 5; 3)$, b) $\overline{AB}=(4; -6; 2)$. **43.** $|\overline{a}|=\sqrt{3}$; $|\overline{b}|=2\sqrt{5}$, $|\overline{c}|=\sqrt{14}$, $|\overline{d}|=\sqrt{30}$. **44.** ± 3. **45.** a) $\overline{a}(3; 6; -3)$, b) $\overline{a}(-3; -6; 3)$. **46.** a) 1 yoki -1; b) 3 yoki -1; c) 2 yoki -4; d) 3 yoki 5/3. **48.** $D(-2; 0; 1)$. **50**. $n=\frac{4}{3}$; $m=\frac{3}{2}$. **52.** a) $D(3; 0; 0)$. **56.** $\overline{c}(-3; -4; 8)$, $|\overline{c}|=\sqrt{89}$; 2) $\overline{c}(4; 5; 5)$, $|\overline{c}|=\sqrt{66}$. **57.** $\overline{c}(-3; 4; 0)$, $|\overline{c}|=5$; 2) $\overline{c}(0; 2; 6)$, $|\overline{c}|=2\sqrt{10}$. **59.** $\overline{a}=\overline{i}-\overline{j}+\overline{k}$, $\overline{b}=2\overline{j}-4\overline{k}$, $\overline{c}=2\overline{i}+3\overline{j}-\overline{k}$, $\overline{d}=\overline{i}+2\overline{j}+5\overline{k}$. **60.** $\sqrt{59}$, $\sqrt{219}$, $\sqrt{122}$, $\sqrt{918}$. **63.** $AC = AO + OC = 4i + 2k$, $AC(-4; 0; 2)$; $CB = CO + OB = 2k + 9j$, $CB\ (0; 9; 2)$; $AB = AO + OB = -4i + 9j$, $AB(-4; 7; 0)$. **65.** $\approx 180N$. **66.** a) 60°; b) 30°; c) 90°; d) 60°; e) 45°. **67.** a) -6; b) 3; c) -6; d) 3. **68.** a) 40°; b) 140°; c) 150°. **69.** a) 30; b) 3; c) 15; d) -28. **70.** a) 1/3; b) -1; c) 2; d) 4. **71.** a) 16. **75.** a) 1; b) 0. **76.** $\overline{BF}=2(\overline{DO}-\overline{DC})$. **77.** $\frac{1}{3}(2\overline{AC}-\overline{AB})$. **78.** $\frac{1}{3}(\overline{AB}+\overline{AC})-\overline{AD}$.

83. a) $(1; -1; 7)$; b) $(-2; 3; 1)$; c) $(0; -4; 4)$. **84.** $\overline{p}(-1; 5; 3)$. **86.** $B(-8; 4; 1)$. **88.** $(2; -5; 9)$; $(-2; -2; 7)$; $(6; -12; 2)$. **93.** Oxz tekislikka nisbatan. **100.** $(0; -3; 1)$. **106.** a) 36 cm; b) 48 cm; c) 6 cm; d) 4 cm. **110.** a) $B(-5; 7,5; 12,5)$; b) $B(5; -7,5; -12,5)$; c) $B(-0,5; 0,75; 1,25)$; d) $B(0,5; -0,75; -1,25)$. **111.** a) $B(-2,5; 1; 3)$; b) $B(-7; 2; 6)$. **112.** a) $O_1(0; 0; 0)$, $A_1(-4; 0; 0)$, $B_1(0; -4; 0)$, $C_1\ (0; 0; -4)$; b) $O_1(-4; 0; 0)$, $A_1(4; 0; 0)$, $B_1(-4; 8; 0)$, $C_1\ (-4; 0; 8)$. **115.** $(2;-3; 3)$. **116.** -3. **117.** $(7; 1; 2)$. **118.** $(1; -2; 3)$. **119.** $(-1; -2; -3)$. **120.** $(1; 2; -3)$. **121.** $(-2; -3; -5)$. **122**. $D(0; 9;-7)$. **123.** $C(2; 0;-8)$. **124.** 19. **125.** $(-7; 7; -7)$. **126.** $(1; 2; 1)$. **127.** $(-2; 7; 1)$. **128.** ± 2. **129.** ± 3. **130.** 13. **131.** 10. **132.** 9. **133.** 0. **134.** -2. **135.** 1. **136.** 4. **137.** 90°. **138.** 4. **139.** -4. **140.** -2; 4. **141.** $8\vec{i}+9\vec{j}-4\vec{k}$.

1- test sinovi javoblari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
C	D	D	B	D	B	B	A	A	D	B	B	B	C	A	C	B	D

1- nazorat ishi javobi

1) $(1; 2; -3)$; 2) 13; 3) $\sqrt{2}$; 4) 90°; 5) 1.

2- bob javoblari

142. 47°, 133°, 47°, 133°. **143.** 128°. **144.** 80°. **145.** 90°. **146.** 5 cm, 5 cm. **147.** 12 cm. **148.** 5 cm. **152.** 45°. **153.** 45°. **154.** 80°. **159.** 60°, 45°. **165.** a) 4, 10; b) 5, 12. **166.** Yo'q. **170.** 6, kub. **171.** 15 ta. **172.** 9 ta. **173.** 180 ta. **174.** 24 cm^2. **175.** 44 cm^2. **176.** 76,8 cm^2. **177.** 17,64 cm. **178.** $4\sqrt{3}cm^2$, 4 cm. **179.** 124 dm^2. **180.** 20 m^2, 30 m^2. **181.** 8 cm, 8 cm. **182.** 13 cm, 9 cm. **184.** 4500 cm^2. **185.** 7,5. **186.** 4. **187.** 480 cm^2. **188.** $5\sqrt{2}$. **189.** 45 cm^2. **190.** 144. **191.** a)18; b)76; c) 110; d) 132; e) 48; f) 96; g) 124. **192.** a) 146; b) 126; c) 108; d) 146. **193.** 84 cm. **194.** $3\sqrt{2}$ cm^2. **195.** 216 cm^2. **196.** a) 58; b) 62; c) 94. **197.** a) 38; b) 92; c) 48. **198.** ≈68 m^2. **199.** 104 cm. **200.** 68 cm^2. **201.** 78 cm^2. **204.** 5120 cm^3. **207.** 144. **209.** 8. **210.** 5. **211.** 6. **212.** 3. **213.** 24. **214.** 2. **215.** 8. **216.** 8. **217.** 72. **218.** 4. **219.** 27 litr. **220.** 4. **221.** 60 cm^2. **222.** $\frac{(S-ab)ab}{4(a+b)}$. **223.** 30 m. **224.** 1200. **225.** a) 4; b) 40; c) 71; d) 88; e) 18; f) 33; g) 78. **226.** a) 90; b) 77; c) 54; d) 96. **227.** 6 m^3. **228.** a) 21; b) 26; c) 58. **230.** 6 m^3. **231.** $\sqrt{2}$ m^3. **232.** $\frac{a^3\sqrt{2}}{2}$. **233.** $2\sqrt{\sin 3\alpha \sin^3\alpha}$. **234.** $abc\sqrt{-\cos 2\alpha}$. **235.** a) $\frac{a^2b\sqrt{3}}{4}$; b) a^2b; c) $\frac{3a^2b\sqrt{3}}{4}$. **237.** 3060 m^3. **238.** 3 cm^3. **239.** $\frac{a^3}{8}$. **240.** $3\sqrt{3}$ m^3. **241.** 1 marta. **243.** 24 cm^3. **245.** 12 cm^3. **246.** 2 cm. **247.** $\frac{ac\sqrt{12a^2-3c^2}}{8}$. **248.** $\frac{h^3\sin\gamma}{2\operatorname{tg}\alpha\operatorname{tg}\beta}$. **249.** 6048 m^3/soat. **250.** 35200 m^3. **251.** 0,5 g/cm^3. **252.** 150 ta. **253.** 42 ta. **254.** 961 ta. **255.** 13 ta. **256.** 90 ta. **257.** 3315 g. **258.** 60 m^2. **259.** 24 ta. **260.** 24 cm^3. **261.** 1927,2 g. **262.** 1927,2 g. **263.** 960 m^3. **264.** 144 g. **265.** 19,3125 g/cm^3. **266.** 440 m^3. **267.** 0,0127 m^3. **271.** $(y+w+z)yx$. **274.** $a:b:c$. **277.** 240π cm^2 , 280π cm^2. **278.** 48 cm^2. **279.** 5 cm. **280.** 128 cm^2. **281.** $\pi Q/4$. **282.** 36π cm^2. **283.** 4π. **284.** 36 cm^2. **285.** 12π. **286.** 64. 6. **287.** 3 dm. **288.** $2\sqrt{34}$ cm. **289.** 3 dm. **290.** 200π, 250π. **291.** 50, 50 +50/π. **292.** 45π cm^3. **293.** 16π cm^2. **294.** 1500 cm^3. **295.** 800 cm^2. **296.** 1000 cm^2. **297.** 5574 cm^2, 1824 cm^2. **298.** 1375π cm^3, 11,375 kg. **299.** 141900 g, 310860 cm^2. **300.** Birinchisining. **301.** 2041 so'm, 15700 cm^2. **302.** 349,45 cm^2, 492 cm^3, 1747 so'm. **303.** 37680 gallon. **304.** 318 gallon. **306.** 3 cm^3. **307.** 4 cm. **308.** 9 m^3. **309.** 1,125. **311.** a) 45π; b) 3,75π; c) 144π. **312.** a) 14π; b) 937,5π. **313.** 4. **314.** 0,25. **315.** 125π. **316.** 4π. **317.** 3. **318.** 8. **319.** 36. **320.** 36. **321.** 24. **322.** ≈30 m^3. **323.** ≈3000 cm^3. **324.** a) ≈1050 cm^2; b) ≈2250 cm^3. **325.** ≈162 kg. **328.** 7. **329.** 4. **330.** 200. **331.** 160. **332.** 4. **333.** 8. **334.** 168. **336.** 1/3. **337.** 1/3. **338.** 144 m^3. **339.** 56 cm^3. **340.** 6. **341.** 2. **342.** 256. **343.** 96. **344.** 91. **345.** 173 π. **346.** 600π. **347.** 20. **348.** 300.

2- test sinovi javoblari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
B	A	D	A	B	A	C	C	A	A	A	C	C	A	A	C	A	D

2- nazorat ishi javoblari

1) 30^0; 2) $2\sqrt{3}+24$; 3) 1513 *l*; 4) 64π cm^3; 5) 35 dm^2, 6,5 dm^3.

Eslatma. *Geometriyaga doir qiyinroq masalalar tartib raqami yulduzcha bilan, uyda bajarish tavsiya qilinayotgan masalalar qizil rangda berilgan.*

Darslikni tuzishda foydalanilgan va qo‘shimcha o‘rganishga tavsiya etilayotgan o‘quv-uslubiy adabiyotlari va elektron resurslar

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MUNDARIJA

Algebra va analiz asoslari: **M.A. Mirzaahmedov, Sh.N. Ismailov, A.Q. Amanov.**

Geometriya: B.Q. Xaydarov.

MATEMATIKA 11

ALGEBRA VA ANALIZ ASOSLARI, GEOMETRIYA

I QISM

O‘rta ta’lim muassasalarining 11-sinfi va o‘rta maxsus, kasb-hunar ta’limi muassasalari o‘quvchilari uchun darslik

1- nashr

Muharrirlar:	N. Gayipov M. Raemov
Texn. Muharrir:	A. Abdusalomov
Kompyuterda sahifalovchi:	S. G‘ofurov

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O‘zbekiston matbuot va axborot agentligi
"O‘ZBEKISTON" nashriyot-matbaa ijodiy uyi
100011, Toshkent sh., Navoiy 30.
Tel.: (371) 244-87-55, 244-87-20